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Source: *The American Economic Review*, May, 1976, Vol. 66, No. 2, Papers and Proceedings of the Eighty-eighth Annual Meeting of the American Economic Association (May, 1976), pp. 94-101

Published by: American Economic Association

Stable URL: <https://www.jstor.org/stable/1817204>

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The Optimal Taxation of Commodities and Income

By DAVID F. BRADFORD AND HARVEY S. ROSEN*

The last few years have seen a resurgence of interest in the old question of how best to raise tax revenue. Roughly speaking, two different problems have been studied. The first is to find a set of commodity taxes that is optimal given certain efficiency and (sometimes) equity considerations. In a second strain of the literature, it is assumed that the revenue system is based upon income rather than commodity taxation, and the problem is to determine the optimal degree of progressivity (or regressivity).^{1,2}

The principal motivation of some writers in the optimal taxation literature seems to be the discovery of fairly simple rules which policy makers actually can implement. Others are more interested in theoretical exploration of the implications of alternative economic assumptions than in developing usable policy recommendations. Practically all the contributions, however, have been quite mathematical and thus inaccessible to many practitioners in the public finance area. The purpose of this

essay is to discuss in a nontechnical way the methodology and principal conclusions of the optimal taxation literature.³

In Sections I and II are discussed the optimal commodity and income tax literatures, respectively. Following this are some observations on the accomplishments of optimal taxation research and on some open questions.

I. Optimal Commodity Taxation

Since the literature contains many and varied derivations of the principal theorems of optimal commodity taxation,⁴ we shall not carry out detailed proofs here. We can point out, however, some of the variations in the way the problem is posed. Most commonly a revenue constraint is taken as a starting point, together with an assumption that the government must use per unit commodity taxes. Thus lump sum taxes are excluded. If x_i is the quantity of the i th good purchased by the household sector from the production sector (negative if the households are net sellers, as in the case of the commodity "leisure"), and T_i is the per unit tax, the revenue constraint is

$$(1) \quad \sum T_i x_i = R,$$

where R is the required revenue level.

The taxes are the difference between the

* U.S. Treasury Dept. and Princeton University, respectively. The authors would like to thank Roger Gordon for useful conversations and Jay Stuart for assistance in gathering material. An extended version of this paper is available upon request to the authors.

¹ There is some overlapping of these strains. For example, A. B. Atkinson and Joseph E. Stiglitz consider the problem of differential commodity taxation in the presence of an income tax.

² Although we shall focus upon these problems in this paper, the optimal tax literature has had a somewhat wider scope. For example, Peter A. Diamond and James A. Mirrlees consider the problem of optimal expenditure along with taxation.

³ Consult Nicholas Stern or Atkinson and Stiglitz for more technical surveys.

⁴ See, for example, Frank A. Ramsey, M. Boiteux, or Agnar Sandmo.

prices, p_i , received by producers and P_i , paid by the consumers,

$$(2) \quad T_i = P_i - p_i.$$

It is frequently assumed that producer prices are fixed, so that by setting taxes we set consumer prices and hence consumer welfare. The problem is then to make the choice of taxes in such a way as to maximize the resulting consumer welfare, i.e., to minimize excess burden.

A typical approach is to assume there to be only one consumer (hence no distribution problem), with a utility function $U(\cdot)$ depending on consumption vector x . Thus the objective might be to choose P (a vector of consumer prices) and p (a vector of producer prices) to

$$(3) \quad \text{Maximize } U(x(P))$$

subject to

$$\sum_i x_i(P)P_i = 0^5$$

and to

$$(4) \quad U_i(x(P)) = aP_i.$$

Conditions (4) are the familiar first order implications of the household's optimization, with a being the Lagrangian multiplier.

More often an indirect utility function $V(P, 0) = U(x(P))$ is used, (the zero argument draws attention to the assumption of no transfer income) because the derivations become very simple when use is made of "Roy's Identity"

$$(5) \quad \frac{\partial V}{\partial P_i} = -x_i(P) \cdot \frac{\partial V}{\partial M},$$

where $\partial V/\partial M$ is the "income" derivative.

Putting these pieces together in any of several sequences leads to the famous

⁵ Paul A. Samuelson uses a somewhat different formulation of the problem and one which has the virtue of emphasizing the resource releasing function of the taxes.

Ramsey result on optimal commodity taxation:

$$(6) \quad \sum_i T_i S_{ik} = bx_k, \quad k = 1, \dots, m$$

where S_{ik} is the derivative of the demand for the i th good with respect to the k th price, other prices and utility being held constant, and b is independent of k . The lefthand side gives an estimate of the change in demand for the k th good which would occur if the taxes were removed. Hence (6) says that the proportional change in demand (thus estimated) should be the same for all commodities—the Ramsey result.

Conditions (6) can also be expressed in terms of elasticities. Probably the most familiar "optimal tax" result is the form which applies when the off-diagonal elasticities are zero. In this case the first order conditions associated with (3) lead to the "inverse elasticity rule":

$$(7) \quad t_r = \frac{d}{E_{rr}}, \quad r = 1, \dots, m.$$

where $t_r = T_r/P_r$, the percentage or ad valorem rate of tax, d is a constant, and E_{rr} is the elasticity of the ordinary (uncompensated) demand function for the k th good. This formula has certainly been of importance in forming economists' intuitions on tax and price regulatory questions. It underlies the notion of charging according to "what the traffic can bear" in transportation, for example, and is the basis for the acceptance on efficiency grounds of such taxes as those on tobacco and alcohol, the demand for which is presumed price inelastic.

An important application of the analysis is to the presumptive case for direct over indirect taxation. The classic Hotelling argument for marginal cost pricing seemed to some to support the conclusion that an "income tax" will involve no efficiency

cost. When it was recognized, however, that the "income" of the tax system is not the "budget level" of the elementary theory of consumer demand, but rather the product of a certain price, the wage, and a demanded quantity (negative net purchase of leisure), the apparent a priori advantage of an income tax was lost. The analyses of W. J. Corlett and D. C. Hague, I. M. D. Little and Milton Friedman to this effect all are applications of the theory of optimal commodity taxation as is neatly shown by Sandmo.

While the extensive subsequent work has shown how difficult it is to sustain *any* simple rules for commodity taxation, the result of the spreading awareness of this work has been to make economists think about tax questions in a new way and to hasten the search for rules which are reasonably robust.

For example, as Stiglitz and Atkinson point out, optimal tax analysis makes it clear that there is no a priori assurance that the income tax is the single best instrument for income redistribution—such "commodity taxes" as are represented by housing subsidies or food stamps might contribute to an optimal program. Michael Boskin notes that, in view of the differences in the observed elasticities of household supply of the two types of labor (husband labor and wife labor), it is probably efficient to tax these "commodities" at different rates. Martin Feldstein (1975) uses the same basic approach to examine the choice between "tax expenditures" and direct expenditure methods of achieving an increase in a specified activity.

A natural question in view of the interpretation of the income tax as a commodity tax is whether taxation of labor only (i.e., uniform taxation of commodities) is appropriate. Not surprisingly, the answer is that it will be so when labor is inelastically supplied. Sandmo shows that this in turn will follow if utility is separable between

leisure and all other goods and homogenous in those goods. Intuitively this separability means that further efficiency cannot be gained by differential taxation of goods that are "related" to leisure. Several writers have noted an important consequence when this result is reinterpreted in an intertemporal context. If utility is a function of consumption and leisure at different dates and separability obtains, then no taxes on interest income should be levied—consumption is the appropriate tax base. This simply illustrates the challenge, implicit in the optimal tax approach, to the widespread acceptance of taxation on the basis of Haig-Simons income which has been emphasized by Feldstein (forthcoming).

While an "income tax" can be regarded as a tax on the sale of labor (negative net purchase of leisure), there is a feature of actual income taxes which is slighted by such a point of view: it is institutionally feasible to assess taxes at different *rates* on different individuals; in particular, progressive taxation of earnings is possible. This means that the income tax and such related taxes as the expenditure tax are potentially important instruments for meeting distributional objectives. We now turn to the studies which consider the trade-off between such distributional objectives and economic efficiency.

II. The Optimal Income Tax Literature

The problem of optimal income taxation has a long history in economics.⁶ However, most of the recent literature stems from a paper published by James Mirrlees in 1971. A natural way to organize our discussion, then, is to summarize Mirrlees' techniques and conclusions, and then view the ensuing literature as an attempt to explain and modify some of his results.

In Mirrlees' model, society is composed

⁶ See especially F. Y. Edgeworth's important contribution.

of individuals who have identical atemporal utility functions in after tax income and leisure. Individuals differ only in their earning abilities (wage per hour). The government must collect an exogenously determined amount of tax revenue. The problem is to find an income tax schedule (tax function) which maximizes the sum of individuals' utilities subject to this revenue constraint.

Using the tools of the calculus of variations to solve the constrained maximization problem, Mirrlees finds that the optimal tax function exhibits marginal tax rates between zero and one, and that when it is operative, part of the population does not work. Although these results may seem weak, they are really quite remarkable given the absence of specific functional forms for the key relationships in the problem.

In order to get more specific results, more specific assumptions must be built into the analysis. Mirrlees assumes that the utility functions are Cobb-Douglas, and considers both log normal and Pareto distributions of earnings abilities. With these assumptions, the following results emerge: (a) the optimal tax function is approximately linear with a negative intercept, and (b) the optimal tax function is characterized by "low" marginal tax rates which *fall* somewhat with income. Atkinson's (1973) interpolations of Mirrlees' results indicate rates in the neighborhood of 20 percent.

Mirrlees was surprised at how low the marginal tax rates were: ". . . I must confess that I had expected the rigorous analysis of income-taxation in the utilitarian manner to provide an argument for high tax rates. It has not done so." (A study by Ray C. Fair in the same year also generated fairly low implied marginal tax rates.) Apparently, those who read the Mirrlees paper also found the low marginal tax rates counterintuitive, for much of the

literature appears to be an attempt to explain them.

One concern was the maximand of Mirrlees' problem, an unweighted sum of individual utilities, which implies that a "util" to a rich individual adds as much to social welfare as a "util" to a poor individual. To what extent would more egalitarian results (i.e., higher marginal tax rates) emerge if a social welfare function were used which weighted the utilities of the rich less than those of the poor? Atkinson and Feldstein (1973) consider social welfare functions of the form

$$(8) \quad W = (\sum U_i^v)^{1/v} \quad v \leq 1.$$

Clearly, when $v=1$, welfare (W) is the simple sum of utilities (U_i). When v is less than 1, however, it can be shown that a given increment to the utility of a low utility individual adds more to W than if awarded to a high utility individual. It should be noted, however, that the specifications of the social welfare function and the individual utility functions are not really independent of each other. We could, for example, specify the utility of the i th individual to be U_i^v and then write social welfare as the arithmetic sum of these utilities.

Atkinson focuses attention on the case in which v approaches minus infinity. Under such circumstances, it can be shown that maximizing W is equivalent to maximizing the utility of the worst off individual in society: the maximin case. This case has received considerable attention due to philosopher John Rawls' argument that it is particularly compelling as an ethical criterion. (A number of criticisms of Rawls' position are suggested by Alvin K. Klevorik.)

Atkinson uses a Rawlsian social welfare function in a model with a linear income tax, no net government revenue requirement (i.e., taxation for redistribution

only), and a Pareto distribution of skills in the economy. He finds that optimal marginal tax rates range between 30 and 45 percent. Thus, one solution to the mystery of Mirrlees' low marginal tax rates is his formulation of the objectives of the government. Social welfare functions which are more egalitarian than the classical utilitarian variety may yield higher marginal rates.

Another potential explanation for Mirrlees' results is the Cobb-Douglas assumption on the form of individuals' utility functions. Stern has investigated this possibility by assuming that individuals have constant elasticity of substitution (*CES*) utility functions in leisure and income. Using results on the elasticity of labor supply from the econometric literature,⁷ he finds that an elasticity of substitution of about .4 is more realistic than 1.0.⁸ When a variant of Mirrlees' problem is solved using *CES* utility functions with this lower elasticity of substitution, the optimal marginal tax rates are substantially higher—without appeal to a more egalitarian social welfare function.

Reexamination of the social welfare function suggests another possible explanation for the low tax rates typically generated by optimal income tax studies. Our intuition about optimal income taxation may perhaps be conditioned on societal objective functions which are not utilitarian-individualistic. For example, the presence in the social welfare function of a variable parameterizing the "aesthetics" of the income distribution would lead to more egalitarian results.⁹ Similarly Feldstein

⁷ These are measures of the elasticity of hours per year with respect to the wage, and thus do not take into account other, perhaps more important, dimensions of labor supply.

⁸ If the elasticity of substitution were zero, lump sum taxation would be possible. If the elasticity of substitution were infinite, no revenue could be raised.

⁹ Such a social welfare would be non-Paretian, but there is nothing to prevent a reasonable set of value judgments from allowing for such a possibility.

(forthcoming) has shown that if interdependent utility functions are allowed for, very high marginal tax rates may be appropriate.

We now turn to a limitation of the Mirrlees model which is just beginning to receive attention, its atemporal setting. The appropriate taxation of capital income is one of the most controversial aspects of the tax system, yet the studies cited above for the most part ignore it. Janusz A. Ordover and Edmund S. Phelps examine the optimal mix of taxes on two factors of production (capital and labor) in a one sector neoclassical growth model.¹⁰ Their model is very general, and therefore no results on tax rates emerge which can be compared to those discussed above. Moreover, the only social welfare function they consider is the maximin case. Despite these limitations, explicit attention to the taxation of capital income in the optimal income tax framework is an important step which will no doubt stimulate further research.

We could continue to list additional aspects of the Mirrlees model which have been changed and expanded in order to ascertain their effects on optimal tax rates.¹¹ However, the basic thrust of the literature should now be clear. An exogenously determined amount of tax revenue must be raised by income taxes on individuals whose economic choices are distorted by the presence of those taxes. Given technological and behavioral assumptions, the optimal tax schedule is that which leaves some social welfare function at a maximum after the tax is collected. The literature shows how various

¹⁰ E. Sheshinski (forthcoming a) considers taxation in a one sector neoclassical growth model with earned and unearned income taxed at the same rate.

¹¹ For example, Stern has suggested changing the assumptions on the underlying distribution of skills, Sheshinski (forthcoming b) focuses attention on a model in which taxes influence human capital accumulation, and Feldstein (1973) allows for an endogenous wage.

assumptions on these components lead to different conclusions regarding the shape of the optimal income tax schedule.

III. Concluding Remarks

The accomplishments of the optimal taxation research have been considerable. It has upset many comfortable rules of thumb and lent precision to many informal arguments. But there remains work to be done. Part of this work will, of course, consist of increasing the stock of variations on the basic problems for which solutions have been described. Another, and very important part will consist of the attempt to determine quantitatively which of these problems best describes the actual economy to be taxed—filling in all those empty boxes with real, estimated elasticities.

However, work of another kind is needed to advance the normative power of the analysis. Normatively the optimal tax literature rests on a utilitarian base. It is true that the optimal commodity tax results, or some of them at least, can be cast, in a form which says: if your tax system doesn't look like this, there is a potential bargain which can be struck among your citizens which would make all better off. However, these bargains are complex and their possibility tends to be eliminated by the very assumptions that require the use of second-best instruments in the first place. For practical application implicit interpersonal utility comparisons are required. The optimal income tax results are also dependent on such comparisons. The missing link is a welfare function, and the question is how does one persuade a legislature or an electorate to decide in accordance with some particular welfare function?

Asking the optimal tax researchers to resolve this is effectively to ask them to make welfare economics persuasive, obviously a tall order. But, interestingly, the tax literature has always appealed to non-

utilitarian criteria as well.

Thus, missing from the optimal tax literature is the idea of horizontal equity, the notion that “. . . people in equal positions should be treated equally.” (Richard A. Musgrave 1959, p. 160) (Customarily, “equal positions” are defined in terms of an observable index of ability to pay such as income, expenditure, or wealth.) In none of the studies discussed above has the injunction to treat equals the same appeared either as a constraint in the maximization problem, or as an argument in the objective function. Therefore, they will in general¹² fail to provide horizontal equity. In light of this, Musgrave (forthcoming) and others have suggested that it is inappropriate to characterize such schemes as “optimal.”

Defining horizontal equity in terms of income is inadequate because individuals with identical opportunity sets but different tastes will have different incomes. An alternative way to define equal position would be identical opportunity sets. However, it seems more in the spirit of the optimal taxation literature to define equal position in terms of utilities: individuals are “the same” only if they derive identical amounts of utility from their consumption and leisure bundles. The choice of a criterion for horizontal equity is important because when tastes differ between individuals,¹ different criteria may lead to different conclusions as to the fairness of a given tax. For example, an income tax which is perfectly fair according to conventional notions of horizontal equity hurts an “income lover” more than a “leisure lover.”

In an attempt to put the discussion of

¹² It can be shown that if all individuals have identical tastes and there is only one type of ability, then horizontal equity will be satisfied by virtually any broad-based tax. (See Feldstein, forthcoming.) Such assumptions, as we have seen, are built into a number of the optimal tax studies. For an exception, see Diamond and Mirrlees.

horizontal equity and the optimal taxation literature on the same plane, Feldstein (forthcoming) has redefined the principle of horizontal equity in terms of utility rather than ability to pay.¹³ However, complete integration of horizontal equity into the optimal tax framework remains to be done. Perhaps this could be accomplished by including some measure of departure from horizontal equity as an argument in the social welfare function, but this approach is bedeviled by conceptual difficulties in measuring departures from horizontal equity.¹⁴

It may well be that horizontal equity, ancient and honorable criterion of tax policy though it be, is not a helpful concept. However, apparent appeal of this nonoperational idea to practical people suggests the attractiveness of properties of a tax structure which are independent of the economy to which that structure is applied. To discover whether there are any such properties which significantly narrow the range of "good" tax structures might be a useful topic of research.

¹³ "If two individuals would be equally well off (have the same utility level) in the absence of taxation, they should be equally well off if there is a tax."

¹⁴ See Rosen for a discussion of these problems and some attempts to surmount them.

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