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The Macroeconomics of the Price Mechanism

Hans Brems

Department of Economics

Lectures to Be Given at the Charles University of Prague

Winter 1991/1992

Bureau of Economic and Business Research
College of Commerce and Business Administration
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MACROECONOMICS

PREFACE

The price mechanism was not always part of macroeconomic theory. It found no room in seventeenth-century or early twentieth-century demand-side macroeconomics. It found ample room in mid-eighteenth and late twentieth-century supply-side macroeconomics.

The present minicourse will derive the repeated reversal of demand-side into supply-side macroeconomics as rigorously and as succinctly as possible. But supply-side macroeconomics was as static as demand-side macroeconomics had been. The closing chapters of the minicourse will dynamize supply-side macroeconomics.

Modern economic theory comes in mathematical form, and no other form will do. The minicourse confines itself to elementary algebra and calculus. A reader needing help will find some in our appendix.

Chapters 1 and 3 are new. Chapter 2 is newly written but based on material published in chapter 15 of my Pioneering Economic Theory 1630-1980, A Mathematical Restatement, Baltimore: Johns Hopkins University Press, 1986.

University of Illinois, September 1991

How much modernist stuff, gone wrong and turned sour and silly, is circulating in our system!

J. M. Keynes, Ec. J. 56, *1946*, *p. 81*

CONTENTS

MACROECONOMICS

CHAPTER 1

STATICS: KEYNESIAN AND MODERN MACROTHEORY

Abstract

Between the last half of the seventeenth century and the mid-eighteenth century macroeconomic theory reversed itself from a demand-side to a supply-side equilibrium. For good measure the reversal repeated itself in the last half of our own century. The paper will derive such a reversal as rigorously and as succinctly as possible. Explicit solutions will be found for the equilibrating variables of a demand-side as well as of a supply-side equilibrium. Sensitivities of solutions to policy instruments will be found and compared.

I. INTRODUCTION

1. Macroeconomics--The Oldest Part of Our Building

Macroeconomics is the oldest part of our building: we have practiced it since the last half of the seventeenth century. Macroeconomics considers an economy producing a single good. Unemployment theory determines the physical output of that good, inflation theory determines its price. Let us take a closer look at their historical origins.

2. Early Demand-Side Equilibria: Unemployment Theory

The concern of the mercantilists was unemployment. Petty [1662 (1899: 30)] estimated it an ten percent and analyzed it within the framework of a demand-side equilibrium. Here physical output was seen as bounded by demand. Supply was no problem: demand would always create its own supply. There was always excess capacity. The rate of interest was determined by the demand for and the supply of money hence could be affected by the money supply. Petty thought that ample money had reduced the rate of interest to six percent. Yarranton [1677 (1854: 38)] believed that the use of paper money would reduce it to four percent. Petty [1662 (1899: 29-31)] also recommended public works "of much labour, and little art." In short: monetary or fiscal policy could raise physical output and employment. Capitalism left to itself might be incapable of utilizing its own resources. Government action was the remedy.[1]

Within less than a century such a demand-side equilibrium was to reverse itself.

3. Early Supply-Side Equilibria: Inflation Theory

Hume's concern was inflation, and he analyzed it within the framework of a supply-side equilibrium. Here physical output was seen as bounded by supply. Demand was no problem: supply would always create its own demand. There was never excess capacity. The rate of interest was determined by saving and investment. As a result, Hume [1752 (1875: 321-322)] and Turgot [1769-1770 (1922: 74-76)] agreed, doubling the money supply would not reduce the rate of interest. Hume realized that doubling the money supply of a not fully monetized economy could widen the scope for division of labor hence expand the goods supply. But doubling the money supply of a fully monetized economy, Hume [1752 (1875: 333)] insisted, would merely double prices. Monetary stimuli would simply generate inflation and fiscal stimuli simply crowding-out. Capitalism was entirely capable of utilizing its own resources. Government action, however well meant, was the problem.[2]

4. Purpose

What interests us is the reversal--all the more so since it occurred twice in three centuries: it repeated itself in our own century. This time the demand-side equilibria of Keynes (1936) and Hansen (1941) reversed themselves into the supply-side equilibria of Friedman (1968), Lucas (1972), and Sargent (1973).

Can we derive the reversal of a demand-side equilibrium, whether Mercantilist or Keynesian, into a supply-side equilibrium, whether vintage Hume or modern? The purpose of the present paper is to show how little it takes to do so--and to do it as rigorously and as succinctly as possible. We shall use the following notation.

5. Variables

$C \equiv$ physical consumption

$D \equiv$ demand for money

$E \equiv$ excess demand in goods market

$I \equiv$ physical investment

$L \equiv$ labor employed

$R \equiv$ tax revenue

$r \equiv$ rate of interest

$w \equiv$ money wage rate

$X \equiv$ physical output

$Y \equiv$ money national income

$y \equiv$ money disposable income

6. Parameters

$A \equiv$ autonomous consumption

$a \equiv$ joint factor productivity

$\alpha, \beta \equiv$ exponents of a production function

$B \equiv$ autonomous investment

$b \equiv$ inducement to invest

$c \equiv$ marginal propensity to consume

$F \equiv$ available labor force

$f \equiv$ inducement to hold speculative money

$G \equiv$ physical government purchase of goods

$J \equiv$ autonomous demand for money

$j \equiv$ propensity to hold transaction money

$\lambda \equiv$ "natural" employment rate

$M \equiv$ supply of money

$S \equiv$ physical capital stock

$T \equiv$ tax rate

The price P of goods will be a parameter in a demand-side equilibrium but a variable in a supply-side equilibrium.

II. DEMAND-SIDE EQUILIBRIUM

1. Demand-Side Equilibrium: Solution

A Keynes-Hansen demand-side equilibrium encompassed two markets, a goods market and a money market, and had two equilibrating variables, physical output and the rate of interest. There was no production function. Price was a parameter shutting off the price mechanism. The equilibrium was a partial one having neither enough markets nor enough equilibrating variables. We write it as follows.

Ignore capital consumption allowances and define national income as the market value of physical output:

$$Y = PX \tag{1}$$

Let tax revenue be:

$$R = TY \tag{2}$$

where $0 < T < 1$.

Define disposable income as national income minus tax revenue:

$$y \equiv Y - R \tag{3}$$

Let consumption be a function of disposable real income:

$$C = A + cy/P \tag{4}$$

where $A > 0$ and $0 < c < 1$.

Let investment be a function of the rate of interest:

$$I = B - br \tag{5}$$

where $B > 0$ and $b > 0$.

Let real demand for money be a function of the rate of interest as well as of physical output:

$$D/P = J - fr + jX \tag{6}$$

where $J > 0$, $f > 0$, and $j > 0$.

Two equilibrating variables, i.e., physical output and the rate of interest will clear the goods and money markets:

$$X = C + I + G \tag{7}$$

$$M = D \tag{8}$$

Solve the system (1) through (8) for physical output and the rate of interest:

$$X = \frac{(A + B + G)f + b(M/P - J)}{bj + [1 - c(1 - T)]f} \tag{9}$$

$$r = \frac{(A + B + G)j - [1 - c(1 - T)](M/P - J)}{bj + [1 - c(1 - T)]f} \tag{10}$$

2. Demand-Side Equilibrium: Policy Conclusions

How sensitive are our demand-side equilibria (9) and (10) to fiscal and monetary policy?

Fiscal-policy instruments are government purchases G and the tax rate T. So take the partial derivatives of (9) and (10) with respect to G:

$$\frac{\partial X}{\partial G} = \frac{f}{bj + [1 - c(1 - T)]f} > 0 \qquad (11)$$

$$\frac{\partial r}{\partial G} = \frac{j}{bj + [1 - c(1 - T)]f} > 0 \qquad (12)$$

So if physical government purchase G is up, so is physical output (9) and the rate of interest (10). The higher rate of interest will discourage investment. Consequently there is some crowding-out. Next take the partial derivatives of (9) and (10) with respect to T. On the latter use (6) with (8) inserted:

$$\frac{\partial X}{\partial T} = - \frac{cfX}{bj + [1 - c(1 - T)]f} < 0 \tag{13}$$

$$\frac{\partial r}{\partial T} = - \frac{cjX}{bj + [1 - c(1 - T)]f} < 0 \tag{14}$$

So if the tax rate T is down, both physical output (9) and the rate of interest (10) are up. Again there is some crowding-out.

The monetary-policy instrument is the money supply M. So take the partial derivatives of (9) and (10) with respect to M:

$$\frac{\partial X}{\partial M} = \frac{b/P}{bj + [1 - c(1 - T)]f} > 0 \tag{15}$$

$$\frac{\partial r}{\partial M} = - \frac{[1 - c(1 - T)]/P}{bj + [1 - c(1 - T)]f} < 0 \tag{16}$$

So if the money supply is up, physical output (9) is up but the rate of interest (10) is down. Now let us reverse our equilibrium.

III. SUPPLY-SIDE EQUILIBRIUM

1. The Natural Supply of Goods

Modern supply-side equilibria added the missing market and the missing equilibrium variable. The missing market was the labor market. Here firms are demanding labor and are facing diminishing returns to it. Let their production function be of Cobb-Douglas form:

$$X = aL^{\alpha}S^{\beta} \tag{17}$$

where $0 < \alpha < 1$, $0 < \beta < 1$, $\alpha + \beta = 1$, and $a > 0$.

Purely competitive firms optimize employment by equating the real wage rate with the physical marginal productivity of labor:

$$\frac{w}{P} = \frac{\partial X}{\partial L} = a\alpha L^{-\beta}S^{\beta} \tag{18}$$

Raise both sides to the power $-1/\beta$ and find demand for labor

$$L = (a\alpha)^{1/\beta}(w/P)^{-1/\beta}S \tag{19}$$

Facing such a demand function, how does labor respond? Friedman's answer (1968) was his "natural" rate of unemployment to which current labor-market literature adds institutional color: Lindbeck and Snower (1986) and Blanchard and Summers (1988) distinguish between "insiders," who are employed hence decision-making, and "outsiders," who are unemployed hence disenfranchised. Let insiders accept the "natural" employment rate λ where $0 < \lambda \leq 1$. In other words, if $L > \lambda F$ insiders will insist on a higher real wage rate. If

$$L = \lambda F \tag{20}$$

they will be happy with the existing one. If $L < \lambda F$ they will settle for a lower one.

The real wage rate insiders will be happy with, given their natural rate λ of employment, might be called the "natural" one. Find it by inserting (20) into (18):

$$\frac{w}{P} = a\alpha(\lambda F)^{-\beta}S^{\beta} \tag{21}$$

At the frozen capital stock S, then, labor can have a β percent higher natural real wage rate by accepting a one percent lower natural rate λ of employment.

May the actual real wage rate differ from the "natural" one (6)? According to New Classicals like Lucas (1972), Sargent (1973), and Sargent-Wallace (1975), with rational expectations agents act as if they knew the structure of the model as well as any systematic monetary policy applied to it. Only random hence unanticipated variations of the money supply can generate deviations of actual from natural. For example let a random hence unanticipated expansion of the money supply encourage demand. Let goods prices respond more readily than does the money wage rate and let employers perceive the response sooner than does labor. At first, then, a real wage rate lower than (21) will be perceived by employers but not yet by labor. As a result, actual employment will exceed the natural one (20). Vice versa, let a random hence unanticipated contraction of the money supply discourage demand. At first, then, a real wage rate higher than (21) will be perceived by employers but not yet by labor. As a result, actual employment will fall short of the natural one (20). But, as Friedman (1968) insisted, eventually labor will perceive and respond: new rounds of collective bargaining will restore the equality between the actual and the natural real wage rate, hence the equality between the actual and the natural employment. Labor has no money illusion.

At the frozen capital stock S the supply of goods corresponding to the natural rate λ of employment may be called the "natural" supply.[3] Find it by inserting (20) into (17):

$$X = a(\lambda F)^{\alpha} S^{\beta} \tag{22}$$

2. Supply-Side Equilibrium: Solution

At this point do we have an overdetermined system? We have two alternative physical outputs X. The first is the physical output (9) matching demand for it. The second is the most profitable physical output (22) at which the real wage rate matches the physical marginal productivity of labor. May the two differ? As long as price P remains frozen they may. If they do, there will be positive or negative excess demand defined as the differences between them:

$$E = \frac{(A + B + G)f + b(M/P - J)}{bj + [1 - c(1 - T)]f} - a(\lambda F)^{\alpha} S^{\beta} \tag{23}$$

Now unfreeze price P, thus allowing excess demand to affect it: let a positive excess demand raise price and a negative excess demand lower it. But there is a feedback: price, in turn, will affect excess demand. Excess demand (23) is a function of price P because demand (9) is, whereas supply (22) is not. Specifically, excess demand (23) is a declining function of price P. To see that it is, take the partial derivative

$$\frac{\partial E}{\partial P} = - \frac{b}{bj + [1 - c(1 - T)]f} \frac{M}{P^2} < 0 \qquad (24)$$

A higher price, then, will lower a positive excess demand and keep lowering it until it has vanished. A lower price will raise a negative excess demand and keep raising it until it has vanished. In short, there ought to be a price at which the market will clear. To find it set (23) equal to zero and solve for P:

$$P = bM/H, \text{ where} \qquad (25)$$

$$H = a(\lambda F)^{\alpha} S^{\beta} \{bj + [1 - c(1 - T)]f\} - (A + B + G)f + bJ$$

Corresponding to any value (25) of P there will be a corresponding value of the money wage rate w satisfying (21) and a corresponding value of the rate of interest found by inserting (25) into (10) and solving for r:

$$r = \frac{A + B + G - a(\lambda F)^{\alpha} S^{\beta}[1 - c(1 - T)]}{b} \tag{26}$$

Policy conclusions drawn from such supply-side equilibria will reverse the policy conclusions drawn from our demand-side equilibria (9) and (10). Let us draw them.

3. Supply-Side Equilibrium: Policy Conclusions

How sensitive are the new supply-side equilibria (22), (25), and (26) to fiscal and monetary policy?

Fiscal-policy instruments are government purchases G and the tax rate T. So take the partial derivatives of (22), (25), and (26) with respect to G:

$$\frac{\partial X}{\partial G} = 0 \tag{27}$$

$$\frac{\partial P}{\partial G} = f\,\frac{P}{H} > 0 \tag{28}$$

$$\frac{\partial r}{\partial G} = \frac{1}{b} > 0 \tag{29}$$

So if physical government purchase G is up, so is price (25) and the rate of interest (26), but physical output (22) is unaffected. The higher rate of interest will discourage investment--but more than it did in the demand-side equilibrium: since physical output (22) is unaffected in the supply-side equilibrium, investment must be down by as much as government purchase is up. The crowding-out is complete. Next take the partial derivatives of (22), (25), and (26) with respect to T:

$$\frac{\partial X}{\partial T} = 0 \tag{30}$$

$$\frac{\partial P}{\partial T} = - cfX \frac{P}{H} < 0 \tag{31}$$

$$\frac{\partial r}{\partial T} = - c \frac{X}{b} < 0 \tag{32}$$

So if the tax rate T is down, both price (25) and the rate of interest (26) are up, but physical output (22) is unaffected.

The monetary-policy instrument is the money supply M. So take the partial derivatives of (22), (25), and (26) with respect to M:

$$\frac{\partial X}{\partial M} = 0 \tag{33}$$

$$\frac{\partial P}{\partial M} = \frac{b}{H} > 0 \tag{34}$$

$$\frac{\partial r}{\partial M} = 0 \tag{35}$$

So if the money supply is up, price (25) is up in proportion, but physical output (22) and the rate of interest (26) are unaffected--as Hume (1752) had said they would be.

IV. SUMMARY AND CONCLUSION

A demand-side equilibrium encompassed two markets, a goods market and a money market, and had two equilibrating variables, physical output and the rate of interest. There was no production function. Price was a parameter shutting off the price mechanism. At that price industry would always produce a physical output matching demand. There was unemployment simply because that demand was insufficient. We have seen such a demand-side equilibrium (9) as a partial one having neither enough markets nor enough equilibrating variables.

A supply-side equilibrium adds the missing market, i.e., a labor market. Here firms demand labor and are facing diminishing returns to it. Consequently their demand for labor (19) is a function of the real wage rate. There is unemployment simply because that real wage rate is too high: facing the demand for labor (19), unions choose a natural rate of employment $0 < \lambda \leq 1$. The labor market doesn't clear! The natural rate λ, in turn, determines a unique natural supply of goods (22).

Such a supply-side equilibrium also adds the missing equilibrating variable, i.e., the price of goods. Resuming its place in macroeconomics, a price mechanism clears the goods market. Demand (9) and natural supply (22) coincide, reversing both fiscal-policy and monetary-policy conclusions.

In a demand-side equilibrium larger government purchases or a tax cut had raised physical output and the rate of interest: crowding-out was incomplete. In a supply-side equilibrium larger government purchases or a tax cut raise price and the rate of interest but leave physical output unaffected: crowding-out is complete.

In a demand-side equilibrium a larger money supply raised physical output and lowered the rate of interest: there was crowding-in. As Hume had observed, in a supply-side equilibrium a larger money supply raises price proportionately but leaves physical output and the rate of interest unaffected: there is neither crowding-out nor crowding-in.

Our supply-side equilibrium was as static as our demand-side equilibrium had been: nothing moved, capital stock remained frozen. To unfreeze capital stock we need a dynamic framework, and we begin with the simplest one we know, the original neoclassical growth model.

FOOTNOTES

[1]Further documentation in Brems (1986: 19-24).

[2]Further documentation in Brems (1986: 33-37).

[3]Hume, to be sure, knew neither unions nor insiders. But if reflecting the equality sign of our $0 < \lambda \leq 1$, i.e., full employment, eighteenth-century institutions would still generate a unique natural supply of goods (22).

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CHAPTER 2

DYNAMICS: NEOCLASSICAL GROWTH

Abstract

Supply-side macroeconomics was as static as demand-side macroeconomics had been. As an introduction to dynamics the present chapter will restate Solow's neoclassical growth model. Out of very few and simple assumptions the model derived a wealth of conclusions none of which was seriously at odds with historical reality: (1) stationary distributive shares, (2) convergence to steady-state growth of output, (3) identical steady-state growth rates of output and capital stock, (4) stationary rate of return to capital, and (5) identical steady-state growth rates of the real wage rate and labor productivity.

I. INTRODUCTION

1. Statics and Dynamics

A static model determines the levels of its variables at a particular time. Its equations contain variables referring to that time but no derivatives with respect to time: no motion can occur.

A dynamic system determines the time paths of its variables and contains derivatives with respect to time: it allows us to see the economy as what it is, i.e., a growing one.

2. The Solow Model

Solow (1956) built the simplest possible model of growth. There was one good with two uses, consumption and investment. An immortal capital stock of that good was the result of accumulated savings under an autonomously given propensity to consume. Solow did not know that halfway through the Second World War Tinbergen [1942 (1959)] had published a similar model with econometric estimates of its parameters for four countries. But he had done it in German behind enemy lines.

II. THE SOLOW MODEL

1. Variables

$C \equiv$ physical consumption

$g_v \equiv$ proportionate rate of growth of variable v

$I \equiv$ physical investment

$L \equiv$ labor employed

$P \equiv$ price of good

$S \equiv$ physical capital stock

$\sigma \equiv$ physical marginal productivity of capital stock

$w \equiv$ money wage rate

$X \equiv$ physical output

2. Parameters

$a \equiv$ joint factor productivity

$\alpha \equiv$ elasticity of physical output with respect to labor employed

$\beta \equiv$ elasticity of physical output with respect to physical capital stock

$c \equiv$ propensity to consume

$F \equiv$ available labor force

$g_v \equiv$ proportionate rate of growth of parameter v

The symbol t is time. All parameters are stationary except a and F whose growth rates are stationary.

3. Definitions

Define the proportionate rate of growth of variable v as

$$g_v = \frac{d\log_e v}{dt} \tag{1}$$

Define investment as

$$I = g_S S \tag{2}$$

4. Goods-Market Clearance

Equilibrium requires output to equal demand for it:

$$X = C + I \tag{3}$$

5. Income and Product Accounting: Product Exhaustion

Let entrepreneurs apply a Cobb-Douglas production function

$$X = aL^{\alpha}S^{\beta} \tag{4}$$

where $0 < \alpha < 1$; $0 < \beta < 1$; $\alpha + \beta = 1$; and $a > 0$.

Let profit maximization under pure competition equalize real wage rate and physical marginal productivity of labor:

$$\frac{w}{P} = \frac{\partial X}{\partial L} = \alpha \frac{X}{L} \tag{5}$$

Define physical marginal productivity of capital stock as

$$\sigma = \frac{\partial X}{\partial S} = \beta \frac{X}{S} \tag{6}$$

Multiply (5) by L and (6) by S, write real wage and profits bills, and find stationary distributive shares:

$$Lw/P = \alpha X \tag{7}$$

$$S\sigma = \beta X \tag{8}$$

Add (7) and (8) and find the slices adding up to the pie:

$$Lw/P + S\sigma = X \tag{9}$$

Assume full employment:

$$L = F \tag{10}$$

6. <u>Consumption</u>

Let consumption be

$$C = cX \tag{11}$$

where $0 < c < 1$.

III. SOLUTIONS

1. Convergence to Steady-State Growth

To solve the system, insert (10) into the production function (4), take the growth rate (1) of the latter, and find

$$g_X = g_a + \alpha g_F + \beta g_S \tag{12}$$

Here, g_a and g_F are parameters but g_S a variable. Use (11), (3), (1), and (2) in that order to express it as

$$g_S = (1 - c)X/S \tag{13}$$

Take the rate of growth (1) of (13), use (12), and express the proportionate rate of acceleration of physical capital stock as

$$g_{gS} = g_X - g_S = \alpha(g_a/\alpha + g_F - g_S) \tag{14}$$

In (14) there are three possibilities: if $g_S > g_a/\alpha + g_F$, then $g_{gS} < 0$. If

$$g_S = g_a/\alpha + g_F \tag{15}$$

then $g_{gS} = 0$. Finally, if $g_S < g_a/\alpha + g_F$, then $g_{gS} > 0$. Consequently, if greater than (15) g_S is falling; if equal to (15) g_S is stationary; and if less than (15) g_S is rising. We conclude that g_S must either equal (15) from the outset or, if it does not, converge to that value.

Insert equation (15) into (12) and find the growth rate of physical output

$$g_X = g_S \tag{16}$$

Take the rate of growth (1) of (6), use (16), and find the growth rate of the physical marginal productivity of capital stock

$$g_\sigma = 0 \tag{17}$$

Take the rate of growth (1) of (5), use (10), (15), and (16), and find the growth rate of the real wage rate and of labor productivity

$$g_{w/P} = g_{X/L} = g_a/\alpha \tag{18}$$

2. Twice the Propensity to Save: Twice the Capital Coefficient

If the propensity to save 1 - c were twice as high, how would a neoclassical model adjust? Rearrange (13) and write it as

$$S/X = (1 - c)/g_s \tag{13}$$

where g_S stands for the solution (15). An economy otherwise equal but with twice the propensity to save 1 - c will at any time have twice the capital coefficient S/X.

3. The Real Wage Rate and the Wicksell Effect

To solve for the real wage rate insert (4) into (5):

$$w/P = \alpha X/L = \alpha a(S/L)^{\beta} \tag{19}$$

Rearrange (13) and divide it by L:

$$S/L = (1 - c)(X/L)/g_s \tag{20}$$

Insert (20) into (19) and find the solution for the real wage rate

$$w/P = \alpha a^{1/\alpha}[(1 - c)/g_s]^{\beta/\alpha} \tag{21}$$

Here is the Wicksell Effect. An economy otherwise equal but with twice the propensity to save 1 - c will, according to (21), have a $2^{\beta/\alpha}$ times higher real wage rate w/P. Wicksell himself [1901 (1934: 164)] expressed his effect: "The capitalist saver is, thus, fundamentally, the friend of labour."

4. Conclusions

The solutions of the neoclassical growth model possessed five important properties: (1) stationary distributive shares; (2) convergence to steady-state growth of output; (3) identical steady-state growth rates of output and capital stock; (4) a stationary rate of return to capital; and (5) identical steady-state growth rates of the real wage rate and labor productivity.

Empirical work by Christensen, Cummings, and Jorgenson (1980), Denison (1967), (1974), Kendrick et al. (1976), Kravis (1959), Kuznets (1971), and Phelps Brown (1973) has found none of the five properties to be seriously at odds with historical reality.

The present chapter has restated the bare bones of the neoclassical growth model. There was only one kind of capital, physical capital: no knowledge capital, no human capital. There was no money, no government, hence no policy handles.

Our last chapter will try to allow for such things.

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CHAPTER 3

DYNAMICS: "NEW" NEOCLASSICAL GROWTH

Abstract

An augmented Solow growth model has three forms of capital stock in it. First a human capital stock of accumulated flows of education. Second a knowledge capital stock of accumulated flows of R & D. Third a conventional capital stock of accumulated flows of physical investment. The paper solves such a model for its levels as well as for its growth rates and discusses the sensitivities of the solutions to monetary and fiscal policy.

I. INTRODUCTION

The original Solow (1956) model had only two factors, labor and physical capital stock. Joint factor ("total factor" or "multifactor") productivity was growing, but its rate of growth was an unexplained residual. The model was a standing invitation to explain the residual.

Recent literature accepted the invitation. Griliches (1973), (1979), (1988) and Lichtenberg-Siegel (1991) saw a knowledge capital stock of accumulated R & D. Its conceptual and econometric problems were discussed by Griliches (1979: 100). A capital stock of knowledge would be a stock of "results ... embodied in people, blueprints, patents, books, and oral tradition." An aggregation of such items would be "quite presumptuous" but perhaps not be all that different from a stock of "'physical' capital which aggregates buildings, planes, computers, and shovels." Kendrick (1976) saw one-half of the 1969 U.S. capital stock as a human capital stock of accumulated education. The most recent estimate of its productivity is Mankiw-Romer-Weil (1990).

Let a Solow model thus augmented produce a single good but make four alternative uses of it. The good may be consumed, it may be invested in knowledge or physical capital stock, or it may be purchased by government and via education be invested in human capital stock. Let's imagine strong cases: let all education be public; let all R & D and physical capital be private; and let all capital stocks be immortal.

The purpose of the paper is to solve such a model for its levels as well as for its growth rates and to discuss the sensitivities of the solutions to the supplies of labor and saving and to monetary and fiscal policy.

II. THE MODEL

1. Variables

A ≡ capital coefficient of knowledge plus physical capital

B ≡ capital coefficient of human capital

b ≡ tax base

C ≡ physical consumption

D ≡ demand for money

E ≡ flow of education

G ≡ government purchase of goods

g ≡ proportionate rate of growth

H ≡ stock of human capital

I ≡ flow of physical investment

J ≡ flow of R & D investment

K ≡ stock of knowledge capital

k ≡ present gross worth of another unit of knowledge capital

κ ≡ marginal productivity of knowledge capital stock

L ≡ labor employed

$P \equiv$ price of good

$R \equiv$ tax revenue

$r \equiv$ before-tax nominal rate of interest

$\rho \equiv$ aftertax real rate of interest

$S \equiv$ stock of physical capital

$s \equiv$ present gross worth of another unit of physical capital

$\sigma \equiv$ marginal productivity of physical capital stock

$v \equiv$ money salary rate

$w \equiv$ money wage rate

$X \equiv$ physical output

$y \equiv$ disposable money income

2. Parameters

$a \equiv$ joint factor productivity

$\alpha, \beta, \gamma, \delta \equiv$ exponents of a Cobb-Douglas production function

$c \equiv$ propensity to consume disposable real income

$F \equiv$ available labor force

$f \equiv$ fraction of government purchase allocated to education

$\lambda \equiv$ "natural" employment rate

$M \equiv$ supply of money

$m \equiv$ reciprocal of the velocity of money

$n \equiv$ number of firms in economy

$T \equiv$ tax rate

All parameters are stationary except a, F, and M whose growth rates are stationary.

3. <u>Definitions</u>

Define the proportionate rate of growth of variable v as

$$g_v = \frac{d\log_e v}{dt} \tag{1}$$

Under immortal capital stocks, investment in education adds to human capital stock, investment in R & D adds to knowledge capital stock, and physical investment adds to conventional capital stock:

$$E = g_H H \tag{2}$$

$$J = g_K K \tag{3}$$

$$I = g_S S \tag{4}$$

4. Firm Output

Regardless of its use let the single good be produced by n identical firms each applying the Cobb-Douglas function

$$X_i = aL_i^{\alpha} H_i^{\beta} K_i^{\gamma} S_i^{\delta} \tag{5}$$

where α, β, γ, and δ are positive proper fractions summing to 1, where a is joint factor productivity, and where the subscript i refers to the ith firm.

5. Firm Demand for Labor

Let labor be hired at the money wage rate w. The ith firm will maximize its aftertax profits by equating the real wage rate with the physical marginal productivity of labor hired: $w/P = \partial X_i/\partial L_i$. Differentiate, rearrange, and write firm demand for labor

$$L_i = \alpha X_i/(w/P) \tag{6}$$

6. Firm Demand for Services of Human Capital

Let services of human capital be hired at the money salary rate v. The ith firm will maximize its aftertax profits by equating the real salary rate with the physical marginal productivity of services hired: $v/P = \partial X_i/\partial H_i$. Differentiate, rearrange, and write firm demand for services

$$H_i = \beta X_i/(v/P) \tag{7}$$

7. <u>Firm Demand for Knowledge and Physical Capital Stocks</u>

The physical marginal productivities of knowledge and physical capital stocks are

$$\kappa_i = \frac{\partial X_i}{\partial K_i} = \gamma \frac{X_i}{K_i} \quad (8)$$

$$\sigma_i = \frac{\partial X_i}{\partial S_i} = \delta \frac{X_i}{S_i} \quad (9)$$

Their marginal-value productivities will then be $\kappa_i P$ and $\sigma_i P$, respectively. Such marginal-value productivities of immortal capital stocks will be marginal net returns taxed at rate T. Let nominal interest expense be tax-deductible, then money may be borrowed at an aftertax nominal rate of interest $(1 - T)r$. Discount future cash flows at that rate. Define present gross worths k_i and s_i of another unit of knowledge or physical capital stock as the present worth at time τ of all its future aftertax marginal-value productivities.

$$k_i(\tau) = \int_\tau^\infty (1 - T)\kappa_i(t)P(t)e^{-(1-T)r(t-\tau)}dt$$

$$s_i(\tau) = \int_\tau^\infty (1 - T)\sigma_i(t)P(t)e^{-(1-T)r(t-\tau)}dt$$

In (20) we shall see that κ_i and σ_i are stationary. But let price be growing at the rate g_p:

$$\kappa_i(t) = \kappa_i(\tau)$$

$$\sigma_i(t) = \sigma_i(\tau)$$

$$P(t) = P(\tau)e^{g_P(t-\tau)}$$

Insert these, define the aftertax real rate of interest as

$$\rho = (1 - T)r - g_P \tag{10}$$

and take the integrals

$$k_i = (1 - T)\kappa_i P/\rho$$

$$s_i = (1 - T)\sigma_i P/\rho$$

Present net worth of another unit of capital stock is its present gross worth minus its price. In our one-good economy that price is P, so

$$k_i - P = [(1 - T)\kappa_i/\rho - 1]P$$

$$s_i - P = [(1 - T)\sigma_i/\rho - 1]P$$

Optimal capital stock is the size of stock at which the present net worth of another unit is zero:

$$(1 - T)\kappa_i = \rho \qquad (11)$$

$$(1 - T)\sigma_i = \rho \tag{12}$$

To find that size insert (8) and (9) into (11) and (12), respectively, and find firm demand for knowledge and physical capital stock

$$K_i = \gamma(1 - T)X_i/\rho \tag{13}$$

$$S_i = \delta(1 - T)X_i/\rho \tag{14}$$

8. <u>Aggregation</u>

Dare we adopt "the analytically convenient setting of 'representative agent models'" criticized by Gordon (1990: 1136)? Let's do it. Facing the same factor prices our n firms will behave alike. Multiply their identical output (5) and factor demand (6), (7), (13), and (14) by n, define $X \equiv nX_i$, $L \equiv nL_i$, $H \equiv nH_i$, $K \equiv nK_i$, and $S \equiv nS_i$, and write aggregate physical output and factor demand:

$$X = aL^{\alpha}H^{\beta}K^{\gamma}S^{\delta} \tag{15}$$

$$L = \alpha X/(w/P) \tag{16}$$

$$H = \beta X/(v/P) \tag{17}$$

$$K = \gamma(1 - T)X/\rho \tag{18}$$

$$S = \delta(1 - T)X/\rho \tag{19}$$

Use (15) to define aggregate physical marginal productivities

$$\kappa \equiv \frac{\partial X}{\partial K} = \gamma\ \frac{X}{K} \equiv \gamma\ \frac{nX_i}{nK_i} = \kappa_i$$

$$\sigma \equiv \frac{\partial X}{\partial S} = \delta\ \frac{X}{S} \equiv \delta\ \frac{nX_i}{nS_i} = \sigma_i$$

so we may remove the i's from (11) and (12). From (18), (19), and (51) we see that

$$g_K = g_S = g_X \tag{20}$$

so κ, κ_i, σ, and σ_i are all stationary.

9. Unions

Facing the aggregate demand for labor (16), how do unions respond? Friedman's answer (1968) was his "natural" rate of unemployment to which current labor-market literature adds institutional color by distinguishing between "insiders," who are employed hence decision-making, and "outsiders," who are unemployed hence disenfranchised. Let insiders accept the natural employment rate λ where $0 < \lambda \leq 1$. The rate λ is natural in the sense that if $L > \lambda F$ insiders will insist on a higher real wage rate. If

$$L = \lambda F \tag{21}$$

they will be happy with the existing one. If $L < \lambda F$ they will settle for a lower one.

10. Income and Product Accounting: Product Exhaustion

With their i's removed insert (11) and (12) into (18) and (19), respectively, and write our factor demands (16) through (19) as distributive shares:

$$Lw/P = \alpha X \tag{22}$$

$$Hv/P = \beta X \tag{23}$$

$$K\kappa = \gamma X \tag{24}$$

$$S\sigma = \delta X \tag{25}$$

Add them and find the slices adding up to the pie:

$$Lw/P + Hv/P + K\kappa + S\sigma = X \tag{26}$$

11. Government: An Inflationary Distortion

Into (11) and (12) with their i's removed insert the definition (10). Insert the result into (26) and write aggregate physical output as

$$Lw/P + Hv/P + (K + S)[r - g_P/(1 - T)] = X \tag{27}$$

The Internal Revenue Service will tax nominal income, so multiply (27) by P, and will tax the full nominal interest income (K + S)Pr. The tax base is then

$$b = Lw + Hv + (K + S)Pr = PX + (K + S)Pg_P/(1 - T) \tag{28}$$

So the tax base will exceed the value PX of aggregate physical output. The excess $(K + S)Pg_p/(1 - T)$ is an IRS inflationary distortion. Tax revenue is tax base <u>times</u> tax rate:

$$R = bT \tag{29}$$

where $0 < T < 1$. As a first approximation let government finance a deficit by increasing the money supply. The government budget constraint then collapses into

$$GP - R = g_M M \tag{30}$$

As another first approximation [Friedman (1959)] let the demand for money be in proportion to the value PX of aggregate physical output but be no function of the rate of interest:

$$D = mPX \tag{31}$$

where $m > 0$. Let the money market clear:

$$M = D \tag{32}$$

Into (30) insert (28), (29), (31), and (32) and see how the IRS inflationary distortion helps financing government purchase:

$$G = g_M mX + TX + (K + S)g_P T/(1 - T) \tag{33}$$

Let the government allocate the fraction f to education:

$$E = fG \tag{34}$$

Into (2) insert (34) and find human capital stock

$$H = fG/g_H \tag{35}$$

12. Consumption

Define aggregate disposable money income as

$$y = PX - R \tag{36}$$

Let consumption be the fraction c of disposable real income:

$$C = cy/P \tag{37}$$

where $0 < c < 1$. Into (37) insert (28), (29), and (36) and see how the IRS inflationary distortion discourages consumption:

$$C = c[(1 - T)X - (K + S)g_p T/(1 - T)] \tag{38}$$

13. Goods-Market Clearance

The single good of our one-good economy was consumed, purchased by government, or invested in physical or knowledge capital. Let the goods market clear:

$$X = C + G + I + J \tag{39}$$

We may now solve our system for its levels as well as for its growth rates.

III. SOLUTIONS

1. Levels

The goods market is cleared by the aftertax real rate of interest. Solve for it by inserting (3), (4), (18), (19), (20), (33), and (38) into (39) and dividing X away:

$$\rho = (\gamma + \delta)(1 - T)/A, \text{ where} \tag{40}$$

$$A = \frac{(1 - c)(1 - T) - g_M m}{(1 - c)g_P T/(1 - T) + g_S} \tag{41}$$

What is the economic meaning of A? Insert (40) into (18) and (19):

$$K = \frac{\gamma}{\gamma + \delta} AX \tag{42}$$

$$S = \frac{\delta}{\gamma + \delta} AX \tag{43}$$

$$K + S = AX \tag{44}$$

So A is simply the capital coefficient of knowledge <u>plus</u> physical capital.

Insert (44) and (33) into (35):

$$H = BX, \text{ where} \tag{45}$$

$$B = f[g_M m + T + Ag_P T/(1 - T)]/g_H \tag{46}$$

So B is simply the capital coefficient of human capital.

Finally insert (21), (42), (43), and (45) into (15) and solve for aggregate physical output

$$X = a^{1/\alpha}\lambda F\left(\frac{\gamma}{\gamma + \delta}\right)^{\gamma/\alpha}\left(\frac{\delta}{\gamma + \delta}\right)^{\delta/\alpha} A^{(\gamma + \delta)/\alpha} B^{\beta/\alpha} \tag{47}$$

Let the market for services of human capital be clearing at whatever human capital stock has accumulated. Then solve for the real salary rate by inserting (45) into (17):

$$v/P = \beta/B \tag{48}$$

Solve for the real wage rate by inserting (21) and (47) into (16):

$$\frac{w}{P} = \alpha a^{1/\alpha}\left(\frac{\gamma}{\gamma + \delta}\right)^{\gamma/\alpha}\left(\frac{\delta}{\gamma + \delta}\right)^{\delta/\alpha} A^{(\gamma + \delta)/\alpha} B^{\beta/\alpha} \tag{49}$$

Solve for price by inserting (31) into (32):

$$P = M/(mX) \tag{50}$$

2. Steady-State Growth

All parameters were said to be stationary except a, F, and M whose growth rates were stationary. In that case differentiate the natural logarithms of our levels (40), (42), (43), (45), (47), (48), (49), and (50) with respect to time and find their steady-state rates of growth:

$$g_{\rho} = 0 \tag{51}$$

$$g_H = g_K = g_S = g_X = g_a/\alpha + g_F \tag{52}$$

$$g_{v/P} = 0 \tag{53}$$

$$g_{w/P} = g_a/\alpha \tag{54}$$

$$g_P = g_M - (g_a/\alpha + g_F) \tag{55}$$

3. Growth Accounting

Has our augmented Solow model explained or at least reduced Solow's unexplained residual? Let's compare the growth accounting of an original and an augmented Solow model. For that we need estimates of α, β, γ, and δ.

Griliches (1988: 14-15) used a production function whose inputs were labor, knowledge capital stock, and physical capital stock. He summarized findings by himself and others by saying that "the estimated elasticity of output with respect to R & D capital tends to lie between .06 and 0.1." Let's use $\gamma = 1/12$.

Mankiw-Romer-Weil (1990) used a production function whose inputs were labor, human capital stock, and physical capital stock. Exponents of each input of 1/3 were "consistent with our empirical results." Let's use $\alpha = \beta = \gamma + \delta = 1/3$, implying $\delta = 1/4$. We summarize:

$$\alpha = 1/3$$

$$\beta = 1/3$$

$$\gamma = 1/12$$

$$\delta = 1/4$$

Now for our comparison.

Collapse our augmented four-factor Solow model into the original two-factor model by classifying human capital as part of labor and knowledge capital as part of capital. Such classification will give us a production function $X = aL^{\alpha + \beta}S^{\gamma + \delta}$ hence a growth account $g_X = g_a + (\alpha + \beta)g_L + (\gamma + \delta)g_S$. Let $g_L = 0.01$. Then a residual growth rate $g_a = 0.0133...$ will make output grow at the same rate as capital stock, i.e., $g_X = g_S = 0.03$. Of that rate, residual growth g_a is 44 percent.

By contrast, the augmented four-factor Solow model has a production function $X = aL^{\alpha}H^{\beta}K^{\gamma}S^{\delta}$ hence a growth account $g_X = g_a + \alpha g_L + \beta g_H + \gamma g_K + \delta g_S$. Now a residual growth rate $g_a = 0.0066...$ will, in accordance with (52), make output grow at the same rate as all capital stock, i.e., $g_X = g_H = g_K = g_S = 0.03$. Of that rate, residual growth g_a is merely 22 percent. The residual has been cut in half!

But with its new α and g_a our four-factor model still yields the same rate of growth (54) of the real wage rate: $g_{w/P} = 0.02$--found by Phelps Brown (1972).

IV. SENSITIVITIES OF LEVELS

1. Sensitivities to the Supply of Labor

Measure the supply of labor by the natural rate of employment $0 < \lambda \leq 1$. Are our levels sensitive to it? Specifically, does labor or anybody else benefit from lowering it?

At the frozen capital stock of a Sargent-Wallace (1975) model labor could have a higher real wage rate at a lower natural rate λ of employment. But our unfrozen capital stocks (42), (43), and (45) are all in direct proportion to physical output (47) hence to λ. A lower λ, then, simply reduces the economy to a lower scale at which factor proportions remain the same. The real wage rate depends upon factor proportions hence remains the same: the natural rate λ is absent from the solution (49). So labor doesn't benefit from the lower λ; nobody benefits. The economy is simply accumulating proportionately less capital stock and producing proportionately less output. The economy is impoverishing itself.

2. Sensitivities to the Supply of Saving

Measure the supply of saving by the propensity 1 - c to save disposable real income. Are our levels sensitive to it? The clue is the capital coefficient A of knowledge <u>plus</u> physical capital.

To see that $\partial A/\partial(1 - c) > 0$ write (41) as

$$A = \frac{1 - T - g_M m/(1 - c)}{g_P T/(1 - T) + g_S/(1 - c)} \tag{41}$$

Here if 1 - c is up, numerator is up, denominator down, and A up. As a result (40), (48), and (50) are down: the aftertax real rate of interest ρ, the real salary rate v/P, and price P are down. But (42), (43), (45), (46), (47), and (49) are up: all capital stocks K, S, and H, physical output X, and the real wage rate w/P are up. There <u>is</u> a Wicksell Effect!

3. <u>Sensitivities to Monetary and Fiscal Policy</u>

Our monetary-policy instrument is the rate of growth g_M of the money supply. Our fiscal-policy instrument is the tax rate T. Are our levels sensitive to such instruments? The clues are the capital coefficients A and B.

In (41) with (55) inserted $\partial A/\partial g_M < 0$: if g_M is up, numerator is down, denominator is up, and A down. $\partial A/\partial T < 0$: if T is up, again numerator is down, denominator up, and A down. A was the capital coefficient of knowledge *plus* physical capital, both assumed to be private. In short: the "private" capital coefficient is always down if g_M or T is up.

In (46) the signs of $\partial B/\partial g_M$ and $\partial B/\partial T$ are not unequivocal. But our appendix finds them to be positive in realistic ranges of g_M and T. B was the capital coefficient of human capital, and all education was assumed to be public. In short: in realistic ranges the "public" capital coefficient is up if g_M or T is up.

Such crowding-out is accomplished *via* an interest mechanism. Write (40) as

$$\rho = (\gamma + \delta) \frac{(1 - c) g_P T/(1 - T) + g_S}{1 - c - g_M m/(1 - T)} \tag{40}$$

and see that if g_M or T is up, numerator is up, denominator down, and ρ up. Private knowledge and physical capital is being crowded out because its cost ρ is up.

May such crowding-out be complete? It may. If g_M or T is up far enough to make (41) reach zero, (40) becomes undefined but has the limit

$$\lim_{A \to 0} \rho = \infty$$

This is Tobin's (1986) "debacle."

Allocation of physical output among capital stocks, then, was sensitive to monetary and fiscal policy. Is the size of physical output also sensitive? The elasticity of physical output (47) with respect to A is $(\gamma + \delta)/\alpha$ and with respect to B β/α. Both capital coefficients A and B are sensitive to g_M and T. So allocation as well as size of physical output are sensitive to g_M or T. We have come a long way since Sargent-Wallace (1975) policy irrelevance. Their capital stock was frozen. Ours--in all three of its forms--is variable.

APPENDIX

The partial derivative of (41) with respect to g_M is

$$\frac{\partial A}{\partial g_M} = -\frac{m + A(1 - c)T/(1 - T)}{(1 - c)g_P T/(1 - T) + g_S} \tag{56}$$

which is always negative. Use it to find the partial derivative of (46) with respect to g_M:

$$\frac{\partial B}{\partial g_M} = f\left[m + \frac{T}{1 - T}\,\frac{Ag_S - g_P m}{(1 - c)g_P T/(1 - T) + g_S}\right]/g_H \tag{57}$$

which is easily positive for realistic values of A, g_S, g_P, and m. Only when g_M becomes very large, hence A very small, will (57) turn negative. In a realistic range, then, B is up if g_M is up.

The partial derivative of (41) with respect to T is

$$\frac{\partial A}{\partial T} = -\ (1 - c)\ \frac{1 + Ag_P/(1 - T)^2}{(1 - c)g_P T/(1 - T) + g_S} \tag{58}$$

which is always negative. Use it to find the partial derivative of (46) with respect to T:

$$\frac{\partial B}{\partial T} = f\left[1 + \frac{g_P}{1 - T}\ \frac{Ag_S/(1 - T) - (1 - c)T}{(1 - c)g_P T/(1 - T) + g_S}\right]/g_H \tag{59}$$

which is easily positive for realistic values of A, g_S, 1 - c, and T. Only when T becomes very large, hence A very small, will (59) turn negative. In a realistic range, then, B is up if T is up.

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APPENDIX: A MATHEMATICAL REMINDER

A MATHEMATICAL REMINDER

Let a and C be constants, u, v, x, and y variables, f and ϕ functional forms, t time, and e Euler's number, the base of natural logarithms.

1. Rules of Differentiation

Chain Rule: $$\frac{df(u)}{dx} = \frac{df(u)}{du}\frac{du}{dx}$$

Constant Rule: $$\frac{da}{dx} = 0$$

Euler's Rule: $$\frac{de^{ax}}{dx} = ae^{ax}$$

Inverse Rule: $$\frac{du}{dx} = \frac{1}{dx/du}$$

Power Rule: $$\frac{dx^a}{dx} = ax^{a - 1}$$

Product Rule: $$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Proportion Rule: $$\frac{d(ax)}{dx} = a$$

Quotient Rule: $$\frac{d(u/v)}{dx} = \frac{v(du/dx) - u(dv/dx)}{v^2}$$

Sum or Difference Rule: $$\frac{d(u \pm v)}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

2. Rule of Integration

The indefinite integral $\int f(x)dx$ of the integrand $f(x)$ will equal $\phi(x) + C$, where C is the constant of integration, if

$$\frac{d\phi(x)}{dx} = f(x)$$

From Euler's Rule of differentiation it then follows that

$$\int e^{ax} dx = \frac{e^{ax}}{a} + C$$

3. Partial Derivatives

Consider a function of more than one variable, say, $u = f(x, y)$. The partial derivatives of that function are

$$\frac{\partial u}{\partial x} \equiv \frac{du}{dx} \text{ treating } y \text{ as a constant}$$

$$\frac{\partial u}{\partial y} \equiv \frac{du}{dy} \text{ treating } x \text{ as a constant}$$

4. The Total Differential

For increments dx and dy the total differential of $u = f(x, y)$ is

$$du \equiv \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$$

5. Natural Logarithms and Rates of Growth

Let $u = e^x$ and $v = e^y$, then their natural logarithms are $\log_e u = x$ and $\log_e v = y$. To such natural logarithms the following rules apply:

Power Rule: $u^a = (e^x)^a = e^{ax}$, hence $\log_e(u^a) = a\log_e u$

Product Rule: $uv = e^x e^y = e^{x+y}$, hence $\log_e(uv) = \log_e u + \log_e v$

Quotient Rule: $u/v = e^x/e^y = e^{x-y}$, hence $\log_e(u/v) = \log_e u - \log_e v$

We have defined the rate of growth g of a variable as the derivative of its natural logarithm with respect to time. Consequently

$$g_{(u^a)} \equiv \frac{d\log_e(u^a)}{dt} = a\frac{d\log_e u}{dt} \equiv ag_u$$

$$g_{(uv)} \equiv \frac{d\log_e(uv)}{dt} = \frac{d\log_e u}{dt} + \frac{d\log_e v}{dt} \equiv g_u + g_v$$

$$g_{(u/v)} \equiv \frac{d\log_e(u/v)}{dt} = \frac{d\log_e u}{dt} - \frac{d\log_e v}{dt} \equiv g_u - g_v$$

In English: the rate of growth of a power of a variable is the exponent <u>times</u> the rate of growth of that variable. The rate of growth of a product is the sum of the rates of growth of its factors. The rate of growth of a quotient is the difference between the rates of growth of its numerator and its denominator.

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His books include:

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