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# Budget-maximizing agencies and efficiency in government 

THOMAS McGUIRE, MICHAEL COINER, and LARRY SPANCAKE*

## 1. Introduction

A recent count for President Carter puts the number of agencies, bureaus and offices within the executive branch of the federal government at $538 .{ }^{1}$ These were responsible for the lion's share of the federal government's \$ 375 billion outlays in fiscal year $1976 .{ }^{2}$ Despite agencies' numerical importance within the national income accounts, public finance theorists de-emphasize their role in determining budget allocations. It is perhaps thought that agencies, as limbs on the executive branch, simply 'do as they're told', exerting no independent influence on public decisions. All of the interesting questions about public goods and services would then center around how demand for public goods is politically expressed. Yet agencies and their officials certainly have their own interests, and may often be in a position to bring influence to bear to advance those interests within the budgetary process. Under this view, a budget is a kind of bargain between elected officials and the agencies rather than a reflection of a unilateral decision by voters' representatives. ${ }^{3}$

Actual bargaining about budgets is a complex, iterative affair involving agency civil servants and political appointees, Cabinet officials, OMB professionals and political appointees, the White House staff, the president, the subcommittees and committees of Congress, and various special interest groups. We try to capture the spirit of the bargaining process in a model of government which relies on the analytical distinction between those parts of government which 'demand' public services and those parts which 'supply' such services.

[^0]Following Niskanen (1971), we designate the 'sponsor' to be the parts of the legislative and executive branches which demand public services. The sponsor deals with 'agencies', which supply public services. ${ }^{4}$ The sponsor maximizes a net benefit function defined over agencies' output and revenue collected. Agencies maximize budgets. ${ }^{5}$ This is a drastic simplification of institutions and motives; but to formally describe a budgetary equilibrium and then to proceed to analyze how that equilibrium changes with respect to changes in the structure of the exchange, it is necessary to begin with stylized institutions and purified motives.

A Civics 101 model of government would have the omniscent sponsor direct agencies to produce services at the rate which maximizes the sponsor's net benefit function subject to the agencies' production possibilities. In this model, the agencies' motives are irrelevant. In our model, the sponsor retains the statutory authority to direct agency production, but is dependent on the agencies to reveal what outputs are possible at what cost.

In the context of our model, we investigate how two factors shape the terms of the budgetary bargain. Our first and major concern is how the structure of agency supply affects budgetary equilibrium. The supply of no public service is 'competitive', but still, there can be competition among agencies. By structure of agency supply, then, we mean not simply the number of agencies producing an identical product, but more generally, the presence of agencies with products substitutes for or complements to an agency's output. Our major result is to demonstrate that competition for the sponsor's funds, even among agencies producing different products, can substantially benefit the ignorant sponsor. The second factor we consider is how information about agencies' cost (allowing the sponsor to do 'cost benefit analysis') helps the sponsor. More information is obviously better for the sponsor than less, but our mjor interesting result here is that information about cost and competitiveness of structure of supply are substitutes in the sense that an increase in one diminishes the contribution to the sponsor's benefit of an increase in the other.

Our interest in the relation between governmental efficiency and the structure of agency supply is shared by the current administration in Washington. President Carter emphasizes the savings from elimination of agencies which duplicate responsibilities of other agencies. Ironically, one target of the Administration's reorganization is the service of anti-trust law enforcement. ${ }^{6}$ The intent is to merge the Bureau of Competition of the FTC (budget: $\$ 23$ million) with the Antitrust Division of the Justice Department (budget: $\$ 26.7$ million), creating a monopoly agency at the Justice Department. There may be some obvious economies to be gained by such a consolidation. Our study emphasizes another consideration: since agencies do not simply 'do as they're told', elimination of an agency increases the bargaining power of agencies producing substitute products. This enhances their ability to advance their own goals at the expense of the
sponsor. Even if some 'waste' is eliminated, the sponsor may be worse off in the new quilibrium.

Section 2 explains the components of our model in more detail and solves the model for the simple case of the sponsor and a single agency. The style of analysis changes markedly in Section 3 where the sponsor faces two agencies. We develop an agency's reaction function to the other's behavior and prove a series of propositions about the relation between agencies' products and the equilibrium net benefit of the sponsor. Sections 4 and 5 use these propositions to analyze the effect on the sponsor of increasing the number of agencies and of informing the sponsor of agencies' cost. Section 6 draws the implications for governmental structure.

## 2. Components of a model of budgeting

The sponsor demands output of public goods and services from agencies ( $a$ to $z$ ) which supply such services. To determine how well this structure of supply serves the sponsor's interest, we need to characterize the sponsor's preferences, the agencies' preferences, and the terms upon which the sponsor and the agencies come together to determine the output of each agency.

### 2.1 Sponsor's preferences

The sponsor's benefit function describes the maximum amount of dollars the sponsor is willing to pay (tax revenue the sponsor is willing to collect) for a combination of outputs of the agencies. Generally, the benefit function is $B(a, \ldots, i, \ldots, z)$ with $z$ agencies. Assume:

$$
B_{i}>0, \text { and } B_{i i}<0, \text { for all } i
$$

Subscripts indicate partial derivatives.

### 2.2 Agencies'preferences

Agency $i$ produces one output at cost $C^{i}(i)$. Agencies maximize the budget they receive from the sponsor subject to covering cost of production. In this model, budget maximization implies productive efficiency; that is, agencies will maximize the output obtainable from any budget. If an agency were not efficient it would be possible to increase production without violating the constraint that the agency cover cost. Since the sponsor's willingness to pay for the agency's output increases at the higher level of output, the agency could propose to the sponsor a higher budget and output which both the sponsor and the agency would prefer to the original, inefficient production. Thus, the agency could not have been maximizing its budget while producing inefficiently. Since an agency's budget is uniquely related to its output by a cost function, output maximization is equivalent to budget maximization for agencies in this model. For analytical convenience, for the rest of this paper, we assume directly that agencies maximize output.

Since agencies always produce efficiently, net benefit to the sponsor can be expressed as a unique function of outputs.

$$
\begin{equation*}
N(a, \ldots, z)=B(a, \ldots, z)-\sum_{i=a}^{z} C^{i}(i) \tag{1}
\end{equation*}
$$

The net benefit function is assumed to be continuous and convex upward. The Hessian $\left[B_{i j}\right]$ is negative definite. This implies that marginal benefits fall faster than marginal cost, and allows substitutability and some complementarity between the outputs. Assume $N(0, \ldots, 0)=0$. Aslo assume $B_{i}(0, \ldots, 0)>C_{i}(0, \ldots, 0)$ for some $i$, so that outputs offering the sponsor positive net benefits are feasible.

### 2.3 Sponsor and agency bargaining

The sponsor and an agency must agree on the budget and output of the agency. In bargaining, as Schelling notes, 'Each party is guided mainly by his expectation about what the other will accept' (1960, p. 20). We assume the agency knows up to what amount the sponsor will pay, if necessary, for any agency output. We assume, initially, the sponsor has no offsetting information about what any output costs the agency ro produce and therefore about the minimum the agency may be willing to accept for an output. Because of the unequal distribution of knowledge, power rests with the agency in the exchange of money for output. In the extreme, the agency maximizes output subject to covering costs and charging no more than the sponsor's maximum willingness to pay. Opportunities to the sponsor are reduced to an all-or-nothing choice. The agency makes an offer which the sponsor may accept or reject. ${ }^{7}$ Later we relax this extreme inequality in information and bargaining power by giving the sponsor some information about cost to the agency. In Section 5, when the sponsor knows a certain output is attainable at a certain cost, the sponsor can simply direct agency behavior.

### 2.4 Inter-agency relations

To define the sponsor's willingness to pay for an agency's output it is necessary to specify how other agencies react to particular offers of the agency in question. This is analogous to the traditional price theory problen of defining uemand for an oligopolist. In spite of its implausibility, to make our problem tractable, we assume, Cournot-like, that in the bilateral dealing between each agency and the sponsor, the behavior of other agencies is regarded as fixed. That is, the sponsor and the agency believe other agencies will not change any offers they have made to the sponsor. An offer is a total output at a total cost. We have assumed that due to its inferior bargaining position the sponsor may only accept or reject an offer. This means that in making an offer to the sponsor, an agency will take other agencies' offers as fixed, although not necessarily the sponsor's acceptance
of those offers. An important characteristic of our model is that although an agency assumes the behavior of other agencies is fixed, an agency recognizes that the sponsor may be induced to reject an offer of another agency it had tentatively accepted.

In spite of what agencies assume, agencies do, of course, respond to one another's offers. Equilibrium will in general be achieved only after numerous rounds of offers, with agencies continually frustrated in their myopic expectations about each other's behavior. This interdependence is not recognized by the agencies. There is no collusion. It must be admitted that such a 'large-number' mentality among the agencies allows the maximum possible competition for a given number of agencies and as such is a polar case. The benefits of interagency competition would be reduced by collusion among agencies.

### 2.5 Equilibrium in a model of budgeting

A set of outputs is a budgetary equilibrium if the following conditions hold: (1) the sponsor cannot increase its net benefit by rejecting a currently accepted offer or by accepting any new offer by the agencies; (2) no agency can increase its output by proposing a new offer that the sponsor would accept. When the sponsor is indifferent between accepting and rejecting an offer we adopt the convention that the sponsor accepts the offer if it is not necessary to reject any other agencies' offers which have already been accepted to remain indifferent. If it is necessary to reject another agency's offer to remain indifferent, the sponsor rejects the new offer. To induce the sponsor to accept an offer when it must reject a currently accepted offer of another agency, we assume the new offer must increment the sponsor's net benefit by a small finite amount $\epsilon$.

### 2.6 Simple case: a single agency

Agency $x$ offers to the sponsor $\bar{x}$ the largest level of output which leaves the sponsor no worse off than with no output at all.

$$
\bar{x}=\max \{x \mid N(x) \geqslant 0\}
$$

This reflects the decisive bargaining power of the agency. Since $N()$ is continuous, the maximum exists and is at the boundary of the constraint set. $\bar{x}$ is defined by (2):

$$
\begin{equation*}
N(\bar{x})=0 \tag{2}
\end{equation*}
$$

The sponsor is in equilibrium at this $\bar{x}$ since the sponsor cannot reject the offer and improve its net benefit. That is,

$$
\begin{equation*}
N(\bar{x}) \geqslant N(0) \tag{3}
\end{equation*}
$$

This equilibrium is inefficient since at $\bar{x}, N(\bar{x})=0$, and marginal benefit is less than marginal cost. There is 'too much' output. The sponsor receives no net benefit from agency production. See Figure $1 .{ }^{8}$


Figure 1. Equilibrium with a single agency.

## 3. Two agencies

As in the move from monopoly to duopoly in models of private good supply, adding a second agency changes and complicates the nature of our model. The sponsor remains passive, accepting or rejecting outputs proposed by the agencies. Agencies, however, may now genuinely respond to one another's offers.

The sponsor accepts an $(x, y)$ if it cannot improve its net benefit by rejecting either or both offers together. That is, for an $(x, y)$ to be acceptable to the sponsor, (4)-(6) must hold:

$$
\begin{align*}
& N(x, y) \geqslant N(0,0)=0  \tag{4}\\
& N(x, y) \geqslant N(x, 0)  \tag{5}\\
& N(x, y) \geqslant N(0, y) \tag{6}
\end{align*}
$$

Agency $x$ maximizes output in reaction to the offer of agency $y$. Agency $x$ assumes agency $y$ 's offer is fixed but it does not assume the sponsor necessarily accepts agency $y$ 's offer. There are thus two strategies for agency $x$ : to offer the maximum $x$ the sponsor would accept based on the sponsor's acceptance of agency $y$ 's offer, and to offer the maximum $x$ the sponsor would accept based on the sponsor's rejection of $y$ 's offer. The largest of these two $x$ 's is agency $x$ 's response to agency $y$. Call the maximum $x$ the sponsor would accept given acceptance of agency $y$ 's offer $F^{x}(y) . F^{x}(y)$ is
the maximum $x$ such that the sponsor accepts $y$, is no worse off than with $y$ alone, and is no worse off than with no output at all.

$$
\begin{equation*}
F^{x}(y)=\max \{x \mid N(x, y) \geqslant N(0, y) \& N(x, y) \geqslant 0\} \tag{7}
\end{equation*}
$$

If a solution to (7) exists, it will be an $x$ for which one of the two inequalities is just satisfied. Since $N(x, y)$ is convex, for any $y$ the solution to $N(x, y)=A$ increase as $A$ decreases. See Figure 2.


Figure 2.
Thus $F^{x}(y)$ is the maximum solution of

$$
N(x, y)=\max \{N(0, y), 0\}
$$

If $N(0, y)>0, F^{x}(y)$ is the maximum solution of

$$
\begin{equation*}
N(x, y)=N(0, y) \tag{7a}
\end{equation*}
$$

If $N(0, y) \leqslant 0, F^{x}(y)$ is the maximum solution of

$$
\begin{equation*}
N(x, y)=0 \tag{7b}
\end{equation*}
$$

Call the maximum $x$ given rejection of agency $y$ 's offer $G^{x}(y) . G^{x}(y)$ is the maximum $x$ such that the sponsor rejects $y$, is $\epsilon$ better off than with $y$ alone, and is no worse off than with no output at all.

$$
\begin{equation*}
G^{x}(y)=\max \{x \mid N(x, 0) \geqslant N(0, y)+\epsilon \& N(x, 0) \geqslant 0\} \tag{8}
\end{equation*}
$$

As in the case of $F^{x}(y)$, if $N(0, y)+\epsilon>0, G^{x}(y)$ is the maximum solution of

$$
\begin{equation*}
N(x, 0)=N(0, y)+\epsilon \tag{8a}
\end{equation*}
$$

If $N(0, y)+\epsilon \leqslant 0, G^{x}(y)$ is the maximum solution of

$$
\begin{equation*}
N(x, 0)=0 \tag{8b}
\end{equation*}
$$

The overall reaction function $x$ to $y$ is $H^{x}(y)$, the maximum of $F^{x}(y)$ and $G^{x}(y)$.

$$
\begin{equation*}
H^{x}(y)=\max \left\{F^{x}(y), G^{x}(y)\right\} \tag{9}
\end{equation*}
$$

Agency $y$ has likewise two strategies in response to an $x, F^{y}(x)$ and $G^{x}(x)$, and an overall reaction function $H^{y}(x)$.

$$
\begin{equation*}
F^{y}(x)=\max \{y \mid N(x, y) \geqslant N(x, 0) \& N(x, y) \geqslant 0\} \tag{10}
\end{equation*}
$$

If $N(x, 0)>0, F^{y}(x)$ is the maximum solution of

$$
\begin{equation*}
N(x, y)=N(x, 0) \tag{10a}
\end{equation*}
$$

If $N(x, 0) \leqslant 0, F^{y}(x)$ is the maximum solution of

$$
\begin{align*}
& N(x, y)=0  \tag{10b}\\
& G^{y}(x)=\max \{y \mid N(0, y) \geqslant N(x, 0)+\epsilon \& N(0, y) \geqslant 0\} \tag{11}
\end{align*}
$$

If $N(x, 0)+\epsilon>0, G^{y}(x)$ is the maximum solution of

$$
\begin{equation*}
N(0, y)=N(x, 0)+\epsilon \tag{11a}
\end{equation*}
$$

If $N(x, 0)+\epsilon \leqslant 0, G^{y}(x)$ is the maximum solution of

$$
\begin{align*}
& N(0, y)=0  \tag{11b}\\
& H^{y}(x)=\max \left\{F^{y}(x), G^{y}(x)\right\} \tag{12}
\end{align*}
$$

An equilibrium pair of outputs, which we call $(\bar{x}, \bar{y})$ is one such that

$$
\begin{align*}
& \bar{x}=H^{x}(\bar{y})  \tag{13}\\
& \bar{y}=H^{y}(\bar{x}) \tag{14}
\end{align*}
$$

and (4)-(6) hold.
The properties and existence of an equilibrium depend on the shape of the sponsor's net benefit function (1), already assumed to be convex upward. In particular, the substituability or complementarity between $x$ and $y$ plays a key role. This relation is defined by the cross partial derivative of the benefit function. ${ }^{9} x$ and $y$ are

$$
\begin{array}{ll}
\text { substitutes iff } & B_{x y}<0 \\
\text { independent iff } & B_{x y}=0 \\
\text { complements iff } & B_{x y}>0
\end{array}
$$

The sign of a second order partial derivative is of course a local property of the benefit function; nevertheless, we assume that the same sign holds for all values of $x$ and $y$ so that the two outputs can unambiguously be classified as substitutes, independent or complements. Since the second partial derivatives are assumed to everywhere have the same sign, and since the cost functions for each of the outputs are independent, we can classify the products, equivalent, as being

$$
\begin{array}{ll}
\text { substitutes iff } & N(x, y)-N(x, 0)-N(0, y)<0 \\
\text { independent iff } & N(x, y)-N(x, 0)-N(0, y)=0 \\
\text { complements iff } & N(x, y)-N(x, 0)-N(0, y)>0 \tag{17}
\end{array}
$$

Using the definiuon of substitutes and complements in (15)-(17), the behavior of the sponsor in (4)-(6) and the behavior of the agencies in (7)-(12), we prove the following propositions:

Proposition 1: If $x$ and $y$ are independent, then an equilibrium exists, is unique, and is stable. $N(\bar{x}, \bar{y})=0$.

Proposition 2: If $x$ and $y$ are complements, then there are many equilibria each with $N(\bar{x}, \bar{y})=0$.

Proposition 3. If $x$ and $y$ are substitutes, $(x, y)$ is an equilibrium if and only if

$$
\begin{aligned}
& N(\bar{x}, \bar{y})=N(\bar{x}, 0) \\
& N(\bar{x}, \bar{y})=N(0, \bar{y}),
\end{aligned}
$$

and

$$
\bar{x} \geqslant x^{*}
$$

$$
\bar{y} \geqslant y^{*},
$$

$$
\begin{aligned}
\text { where } & x^{*}=\left\{x \mid N_{x}(x, 0)=0\right\} \\
y^{*} & =\left\{y \mid N_{y}(0, y)=0\right\}
\end{aligned}
$$

(Subscripts indicate partial derivatives.) This equilibrium is unique and stable. $N(\bar{x}, \bar{y})>0$.

Proposition 4: In equilibrium, $N(\bar{x}, \bar{y})>0$ only if $x$ and $y$ are substitutes.
We now prove each proposition in turn.
Proposition 1: If $x$ and $y$ are independent, then an equilibrium $(\bar{x}, \bar{y})$ exists, is unique, and is stable. $N(\bar{x}, \bar{y})=0$.

Proof: Consider agency $x$ 's reaction to an arbitrary value of $y$ for which $N(0, y)>0 . F^{x}(y)$ is the maximum solution of $(7 \mathrm{a}) . G^{x}(y)$ is the maximum solution of (8a). When $x$ and $y$ are independent, (16) holds, so (7a) can be rewritten as

$$
\begin{equation*}
N(x, 0)=0 \tag{18}
\end{equation*}
$$

It is now possible to compare the maximum $x$ 's which satisfy (7a) (now (18) ) and (8a). Since (8a) requires $N(x, 0)>0$ and since $N_{x}(x, 0)<0$ in this region, we have $F^{x}(y)>G^{x}(y)$, so that $H^{x}(y)$ is the maximum solution to (18).

Consider $H^{x}(y)$ for $y$ such that $N(0, y) \leqslant 0$. Now $F^{x}(y)$ is the maximum solution to (7b) and $G^{x}(y)$ the maximum solution to (8b). Equation (16) and $N(0, y)<0$ imply that the solution to $(7 \mathrm{~b})$ has the property

$$
\begin{equation*}
N(x, 0)>0 \tag{19}
\end{equation*}
$$

Since (7b) requires $N(x, 0)>0$ and since $N_{x}(x, 0)<0$ in this region, we have $G^{x}(y)>F^{x}(y)$, so that $H^{x}(y)$ is the maximum solution to ( 8 b ). Equation (8b) is of course identical to (18).

Thus for any $y$, and this should not be surprising since $x$ and $y$ are independent, $H^{x}(y)$ is the maximum $x$ which solves an equation, ( 8 b ), which does not depend on $y$. Likewise, agency $y$ 's reaction, $H^{y}(x)$ is the maximum $y$ solving (11b). Reaction functions and the isoquant $N(x, y)=0$ are shown in Figure 3. The isoquant is defined by (16).

Since $N(x, y)$ is continuous, solutions to (8b) and (11b) exist. The maximum solutions are unique. $(\bar{x}, \bar{y})$ is trivially stable since for any $x, H^{y}(x)=$ $\bar{y}$ and $H^{x}\left(H^{y}(x)\right)=\bar{x}$. Adding the definition of independence (16) to (8b) and (11b) tells us that the sponsor receives no net benefit in equilibrium,


Figure 3. Reaction functions, $x$ and $y$ independent.
$N(\bar{x}, \bar{y})=0$. The vertical reaction function for agency $x$ crosses the horizontal reaction for agency $y$ at an $(\bar{x}, \bar{y})$ on the isoquant $N(\bar{x}, \bar{y})=0$ as shown in Figure 3. Conditions (4)-(6) for the sponsor's equilibrium are satisfied as equalities.

Proposition 2: If $x$ and $y$ are complements, then there are many equilibria each with $N(\bar{x}, \bar{y})=0$.

Two agencies are truly interdependent when $x$ and $y$ are complements. Figure 4 a and 4 b show reaction functions and selected isoquants of $N(x, y)$. Two figures are necessary since the reaction function may take one of two forms. Since $N(x, y)$ is convex upward, the set of $(x, y)$ such that $N(x, y)$ is greater than some value is closed and bounded. ${ }^{10}$ Isoquants are thus continuous and enclose convex regions. When $x$ and $y$ are complements isoquants are 'stretched' in the NE to SW direction. Rewriting the definition of complements, (17), shows why this is so: $N(x, y)>N(x, 0)+$ $N(0, y){ }^{11}$

Proof: Reaction functions are made up of segments. These segments are defined with the aid of the following particular values of $x$ and $y$.

$$
\begin{aligned}
& x^{1}=\max \{x \mid N(x, 0) \geqslant 0\} \\
& x^{2}=\max \left\{x \mid N\left(x, y^{1}\right)=0\right\} \\
& x^{3}=\max \left\{x \mid N\left(x, y^{4}\right)=0\right\}
\end{aligned}
$$

$$
\begin{aligned}
& x^{4}=\max \{x \mid N(x, y) \geqslant 0\} \\
& y^{1}=\max \{y \mid N(0, y) \geqslant 0\} \\
& y^{2}=\max \left\{y \mid N\left(x^{1}, y\right)=0\right\} \\
& y^{3}=\max \left\{y \mid N\left(x^{4}, y\right)=0\right\} \\
& y^{4}=\max \{y \mid N(x, y) \geqslant 0\}
\end{aligned}
$$

$H^{x}(y)$ is built up by considering agency $x$ 's response to $y$ in various regions of $y$. All this is shown in Figures 4 a and 4 b , and summarized in Table 1. Suppose first that $0 \leqslant y \leqslant y^{1}$ so that $N(0, y)>0 . F^{x}(y)$ is the maximum solution of (7a) and $G^{x}(y)$ the maximum solution of (8a). From (17), the definition of complements, (7a) implies a solution for $x$ such that $N(0, x)$ $<0$. Thus in this region $F^{x}(y)>G^{x}(y)$, so that $H^{x}(y)$ is the maximum solution to (7a). In Figures 4 a and 4 b this is curved line $c d$.

Now suppose $y^{1} \leqslant y \leqslant y^{4}$. Here $N(0, y) \leqslant 0$, so $F^{x}(y)$ is the maximum solution of ( 7 b ) and $G^{x}(y)$ is the maximum solution of ( 8 b ). The maximum $x$ which solves (8b) is $x^{1}$, so $H^{x}(y)$ can be no less than $x^{1}$ for $y$ in this


Figure 4a. Reaction functions, $x$ and $y$ complements, $x^{3}<x^{1}$.


Figure $4 b$. Reaction functions, $x$ and $y$ complements, $x^{3}>x^{1}$.
range. At $y^{1}$, the beginning of the range, (7b) yields $x^{2}$ which must be greater than $x^{1}$ since $x$ and $y$ are complements. From $x^{2}, H^{x}(y)$ increases to $x^{4}$ as $y$ increases to $y^{3}$. It must increase monotonically since $N(x, y)=$ 0 encloses a convex region. Thus from $y^{1}$ to $y^{3} H^{x}(y)$ is the maximum solution (7b). After $y^{3}$, when $H^{x}(y)$ falls, the maximum solution to ( 7 b ) may be less than $x^{1}$. At $y^{4}$, (7b) determines $x^{3}$. If $x^{3} \geqslant x^{1}$ as in Figure 4 a , $H^{x}(y)$ is the maximum solution of (7b), throughout the range $y^{1} \leqslant y \leqslant y^{4}$. If $x^{3}<x^{1}$ as in Figure 4 b , for some $y^{3}<y \leqslant y^{4}, H^{x}(y)=x^{1}$.

Lastly suppose $y>y^{4} . N(0, y)<0$ but there is no solution to (7b) so $H^{x}(y)$ is the maximum solution of (8b). Thus $H^{x}(y)=x^{1}$ when $y>y^{4}$. Table 1 summarizes $H^{x}(y)$.

Table 1. $H^{x}(y)$.
$\overline{H^{x}(y), \text { the maximum value of } x}$
for which

| $0 \leqslant y<y^{1}$ | $N(x, y)=N(0, y)$ |
| :--- | :--- |
| $y^{1} \leqslant y \leqslant y^{3}$ | $N(x, y)=0$ |
| $y^{3}<y \leqslant y^{4}$ | $N(x, y)=0$ or $x=x^{1}$ |
| $y^{4}<y$ | $x=x^{1}$ |

Construction of $H^{y}(x)$ is a symmetric exercise. In both figures, $y^{3}>y^{1}$. It is impossible for both $x^{3}<x^{1}$ and $y^{3}<y^{1}$. If this were true, $N\left(x^{1}, y^{1}\right)$ $<0$ and $x$ and $y$ could not be complements. Thus recognizing the symmetric nature of $x$ and $y$, Figures 4 a and 4 b shows both possibilities for reaction functions when $x$ and $y$ are complements.

Consider now the regions in which equilibria are possible. There is no possible equilibrium where $0 \leqslant y<y^{1}$ since there is no $(x, y)$ for which $H^{x}(y)=x, H^{y}(x)=y$, and $N(x, y) \geqslant 0$ when $x$ and $y$ are complements.

Equilibria are possible when both $H^{x}(y)$ and $H^{y}(x)$ are characterized by $N(x, y)=0$. For a given $y$ it is the maximum $x$ satisfying $N(x, y)=0$ which describes agency $x$ 's reaction. Likewise the maximum $y$ solving $N(x, y)$ is the reaction to $x$. Pairs which lie on both reaction functions, that is, are both the maximum $x$ for a given $y$ and maximum $y$ for a given $x$, are in the northeast portion of $N(x, y)=0$.

Letting ( $\bar{x}, \bar{y}$ ) denote equilibrium values, when $x^{3} \geqslant x^{1}, x^{3} \leqslant \bar{x} \leqslant x^{4}$ and $y^{3} \leqslant \bar{y} \leqslant y^{4}$. These equilibria are points between $e$ and $f$ in Figure 4a. There must be such points since isoquants enclose regions closed and bounded. When $x^{3}<x^{1}$ as in Figure $4 \mathrm{~b}, x^{1} \leqslant \bar{x} \leqslant x^{4}$, and $y^{3} \leqslant \bar{y} \leqslant y^{2}$. In both cases, $N(\bar{x}, \bar{y})=0$.

Some equilibrium is always the final resting point of any $(x, y)$ sequence. Begin with an $x>x^{4}$ in Figure 4a. $H^{x}\left(H^{y}(x)\right)=x^{2}$. For any $0 \leqslant x<x^{3}$, $x<H^{x}\left(H^{y}(x)\right) \leqslant x^{4}$. And for any $x^{3} \leqslant x \leqslant x^{4}, H^{x}\left(H^{y}(x)\right)=x$. Thus all sequences of $x$ go to an equilibrium value, and the same is true for $y$. A similar argument can be made in the case when $x^{3}<x^{1}$ in Figure 4b.

Proposition 3: If $x$ and $y$ are substitutes, $(\bar{x}, \bar{y})$ is an equilibrium if and only if:

$$
\begin{align*}
& N(\bar{x}, \bar{y})=N(\bar{x}, 0)  \tag{20}\\
& N(\bar{x}, \bar{y})=N(0, \bar{y}) \tag{21}
\end{align*}
$$

and

$$
\begin{aligned}
& \bar{x} \geqslant x^{*}, \bar{y} \geqslant y^{*}, \text { where } \\
& x^{*}=\left\{x \mid N_{x}(x, 0)=0\right\} \\
& y^{*}=\left\{x \mid N_{y}(0, y)=0\right\}
\end{aligned}
$$

The equilibrium is unique and stable. $N(\bar{x}, \bar{y})>0$.
Proof: Suppose there is an $(\bar{x}, \bar{y})$ such that (20) and (21) hold and such that $\bar{x} \geqslant x^{*}$ and $\bar{y} \geqslant y^{*}$. From the definition of substitutes, (15), $N(\bar{x}, \bar{y}), N(\bar{x}, 0)$
and $N(0, \bar{y})$ must be positive. Conditions for equilibrium for the sponsor, (4)-(6) are satisfied at $(\bar{x}, \bar{y})$.

Consider agency $x$ 's reaction to $\bar{y}$. Since $N(0, \bar{y})>0, F^{x}(y)$ is the maximum solution of (7a) and $G^{x}(y)$ is the maximum solution of (8a). Since (20) and (21) hold, (7a) implies $F(\bar{y})$ is the maximum $x$ which satisfies (22).

$$
\begin{equation*}
N(x, 0)=N(0, \bar{y}) \tag{22}
\end{equation*}
$$

When (22) has a solution, there are generally two, one $x$ larger and one $x$ smaller than $x^{*}$. The larger $x$ is $F^{x}(\bar{y})$. Since $\bar{x} \geqslant x^{*}, \bar{x}=F(\bar{y})$. And since $N(0, \bar{y})$ from (22) is less than $N(0, \bar{y})+\epsilon$ from (8a), $F^{x}(\bar{y})>G^{x}(\bar{y})$, so $H^{x}(\bar{y})=\bar{x}$. Thus agency $\bar{x}$ reacts to $\bar{y}$ with $\bar{x}$. For similar reasons, $H^{y}(\bar{x})=$ $\bar{y} .(\bar{x}, \bar{y})$ is an equilibrium.

Suppose ( $\bar{x}, \bar{y}$ ) is an equilibrium. Equations (4)-(6) then hold. Also, $H^{x}(\bar{y})=F^{x}(\bar{y})$ and $H^{y}(\bar{x})=F^{y}(\bar{x})$. That is, each agency must be maximizing its output by making an offer based on the sponsor's acceptance of the other agency's offer. If this were not true, one of $\bar{x}$ or $\bar{y}$ would then be rejected by the sponsor and ( $\bar{x}, \bar{y}$ ) would not have been an equilibrium. Thus either of (7a) or (7b) describes $H^{x}(\bar{y})$ and either of (10a) or (10b) describes $H^{y}(\bar{x})$. We show by contradiction that (7b) and (10b) cannot hold. If they did (5) and (6) then imply that each of $N(\bar{x}, 0)$ and $N(0, \bar{y})$ would be less than or equal to zero. But then $x$ and $y$ would not be substitutes (check (15) ). Therefore (7a) and (10a), which at ( $\bar{x}, \bar{y}$ ) are just (20) and (21), must hold.

If $(\bar{x}, \bar{y})$ is an equilibrium, $H^{x}(\bar{y})=\bar{x}$. We show by contradiction that $\bar{x}$ cannot be less than $x^{*}$. If $\bar{x}$ were less than $x^{*}$, agency $x$ could make an offer to the sponsor, such as $x^{*}$, which would give the sponsor high net benefit than $(\bar{x}, \bar{y})\left(\right.$ since $\left.N(\bar{x}, \bar{y})=N(\bar{x}, 0)<N\left(x^{*}, 0\right)\right)$ and increase output of $x$. So if $\bar{x}<x^{*}, H^{x}(\bar{y})>\bar{x}$, and $(\bar{x}, \bar{y})$ would not be an equilibrium. Therefore $\bar{x} \geqslant x^{*}$, and similarly $\bar{y} \geqslant y^{*}$.

We turn now to the properties of the equilibrium $(\bar{x}, \bar{y})$. It was noted at the beginning of this proof that $N(\bar{x}, \bar{y})>0$. We show the equilibrium $(\bar{x}, \bar{y})$ is unique. Rewriting (20) and (21), $(\bar{x}, \bar{y})$ is a solution to

$$
\begin{align*}
& N(x, y)=N(x, 0)  \tag{23}\\
& N(0, y)=N(x, 0) \tag{24}
\end{align*}
$$

Writing out the net benefit function and taking total differentials, we have from (23):

$$
\begin{equation*}
\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{B_{x}(0, y)+\left[C_{x}(x)-B_{x}(x, 0)\right]}{B_{y}(x, 0)+\left[C_{y}(y)-B_{y}(0, y)\right]} \tag{25}
\end{equation*}
$$

The expressions in brackets in (25) are marginal costs less marginal benefits, a difference greater than or equal to zero when $x \geqslant x^{*}$ and $y \geqslant y^{*}$. Both top and bottom in (25) are positive. From (24) we have

$$
\begin{equation*}
\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{B_{x}(x, 0)-B_{x}(x, y)}{B_{y}(x, y)-B_{y}(x, 0)-C_{y}(y)} \tag{26}
\end{equation*}
$$

The slope of (24) described in (26) is negative when $x$ and $y$ are substitutes. Together (25) and (26) tell us that there is at most one intersection of (23) and (24) when $x \geqslant x^{*}, y \geqslant y^{*}$ and $x$ and $y$ are substitutes. Thus if $(\bar{x}, \bar{y})$ exists, it is unique.

Stability of $(\bar{x}, \bar{y})$ is demonstrated with the aid of reaction functions. It is necessary again to describe reaction functions piece by piece. For this purpose define the following values for $x$ and $y$. See Figure 5.

$$
\begin{aligned}
& x^{1}=\left\{x \mid N(x, 0)=N(x, y) \& x<x^{*}\right\} \\
& x^{2}=\max \{x \mid N(x, 0) \geqslant 0\} \\
& y^{1}=\left\{y \mid N(0, y)=N(\bar{x}, \bar{y}) \& y<y^{*}\right\} \\
& y^{2}=\max \{y \mid N(0, y) \geqslant 0\}
\end{aligned}
$$



Figure 5. Reaction functions, $x$ and $y$ substitutes, equilibrium exists.

Consider $H^{x}(y)$ when $x>x^{2} . N(0, y)<0$ so $F^{x}(y)$ is the maximum solution of ( 7 b ) and $G^{x}(y)$ is the maximum solution of ( 8 b ). If a solution to (7b) exists, since $x$ and $y$ are substitutes, it must be an $x$ such that $N(x, 0)$ $>0$. Then, $G^{x}(y)>F^{x}(y)$ in this region, and $H^{x}(y)=x^{2}$. See Figure 5.

Consider $H^{x}(y)$ when $\bar{y}<y \leqslant y^{2} . N(0, y)>0$. So $F^{x}(y)$ is the maximum solution of (7a) and $G^{x}(y)$ is the maximum solution of (8a). Since at $\bar{y}, \bar{x}$ makes no net contribution to net benefits (i.e. (21) holds), and since $x$ and $y$ are substitutes, for a $y>\bar{y}, \bar{x}$ makes negative contribution to net benefit.

$$
\begin{equation*}
N(\bar{x}, y)<N(0, y) \tag{28}
\end{equation*}
$$

The maximum solution of (7a) is an $x$ for which (28) is an equality. In order to increase the left hand side of $(28), F^{x}(y)$ may be greater than $\bar{x}$. Regarding (8a), since $N(0, y) \geqslant N(0, y) \geqslant N(0, \bar{y}), G^{x}(y)$ must be less than or equal to $\bar{x}$. Thus $F^{x}(y)>G^{x}(y)$ in this range and $H^{x}(y)=F^{x}(y)>\bar{x}$. This is shown as the line connecting $(\bar{x}, \bar{y})$ to point $c$ in Figure 5.

Consider $H^{x}(y)$ when $0 \leqslant y<y^{1} . N(0, y)>0$ so $F^{x}(y)$ is the maximum solution of (7a) and $G^{x}(y)$ is the maximum solution of (8a). It is not possible to say in general which of these determines a larger $x .{ }^{12}$ We do know, however, from the reasoning in the last paragraph that $H^{x}(y)>\bar{x}$. It is inconsequential to the model which of (7a) and (8a) hold. In Figure 5, we have drawn the reaction function in this range assuming (7a) describes $H^{x}(y)$.

Figure 5 shows $H^{x}(y)$ and $H^{y}(x)$ for all regions of $x$ and $y$. For $x<\bar{x}$, $\bar{x}<H^{x}\left(H^{y}(x)\right)<x^{2}$. For $\bar{x}>x^{2}, x<H^{x}\left(H^{y}(x)\right)<x^{2}$. Consider a sequence of $x$ 's beginning with an $\bar{x}<x<x^{2} . \bar{x}$ is a lower bound to that sequence since there is no $x$ for which $H^{x}\left(H^{y}(x)\right)<\bar{x}$. Since $N\left(H^{x}\left(H^{y}(x)\right)\right.$, $0)>N(x, 0)$, and $\bar{x}>x^{*}, H^{x}\left(H^{y}(x)\right)<x$. Thus any sequence beginning with $\bar{x}<x<x^{2}$ is decreasing with a lower bound of $\bar{x}$. It must therefore converge to $\bar{x} .(\bar{x}, \bar{y})$ is stable.

Without going into much detail, it is possible to sketch the workings of the model when there is no equilibrium. This can only occur when $x$ and $y$ are substitutes and one of the two sets of conditions in Proposition 3 is not satisfied. When there is no equilibrium there are cycles in outputs. The difficulty is that in the region $\bar{y}<y \leqslant y^{2}$, in each case, reductions in outputs chronicled by reaction function (8a) for agency $x$ and (11a) for agency $y$ lead one agency to offer the output which maximizes the net benefit the sponsor receives from that agency alone before an ( $\bar{x}, \bar{y}$ ) satisfying (20) and (21) is reached. Suppose agency $x$ offers $x^{*}$. Now, in response to $H^{y}\left(x^{*}\right)$ described by (11a), there is no solution $G^{x}\left(H^{y}\left(x^{*}\right)\right)$ to (8a) for agency $x$. That is, there is no offer agency $x$ can make to increase the sponsor's net benefit. Agency $x$ is forced to respond with the $x$ from (7a) which will generally be much smaller than $x^{*}$. To this much reduced $x$, agency $y$
responds with an offer described by (10a) greater than $H^{y}\left(x^{*}\right)$. To this $y$ greater than $H^{y}\left(x^{*}\right)$, agency $x$ can respond with an $x$ from (8a), and the cycle has begun again.

As is typical in Cournot-like duopoly models, our equilibrium concept that each agency maximizes taking the other agency's offer as fixed - is probably more appealing than the dynamics of agency and sponsor behavior out of equilibrium. In moving towards equilibrium each agency's assumption about behavior of its rival is continually contradicted by the other agency's changing behavior. A further difficulty is that in the dynamics of all three cases described above, the sponsor can be led from positions of positive net benefits to positions with lower net benefits.

There is no easy change to make in the model so that agencies behave in a more reasonable way. It is possible, however, to make the sponsor more sensible and less passive by giving the sponsor a 'memory', so that the sponsor does not permit an offer currently being made by an agency to be withdrawn and replaced by an offer which gives the sponsor less net benefits than the original situation.

This eliminates cycling in the models just discussed. Equilibrium for the sponsor with this change is then at a level of net benefits equal to the maximum net benefits from the single agency which can offer the least net benefits alone. If the sponsor has a memory, many more equilibria exist for all possible relations between $x$ and $y$, depending on the starting value of net benefits. An important point about this is that the sponsor never does any better than this initial value for net benefits when $x$ and $y$ are independent or complements. The reason for this is given in discussion of Proposition 4 below.

Proposition 4: In equilibrium $N(\bar{x}, \bar{y})>0$ only if $x$ and $y$ are substitutes.
Proof: This follows directly from Propositions 1, 2, and 3, noting that independence, complementarity, and substitutability represent an exhaustive classification of the relation between $x$ and $y$.

The connection between substitutability of $x$ and $y$ and positive net benefits to the sponsor goes beyond Proposition 4, a statement about equilibria. Out of equilibrium, only when $x$ and $y$ are substitutes does 'competition'between agencies increase net benefits to the sponsor. If $x$ and $y$ are independent or complements and the level of net benefits to the sponsor is greater than or equal to zero, agencies never react within an offer which increases the sponsor's net benefits. Derivation of the reaction functions shows this to be true. Only when $x$ and $y$ are substitutes does agency $x$ respond with an offer described by the maximum solution of (8a), augmenting the sponsor's net benefit by amount $\epsilon$.

It is also important to recognize in this connection that the more sub-
stitutable are $x$ and $y$, the higher the sponsor's net benefits will be. Consider the equilibrium ( $\bar{x}, \bar{y}$ ) when $x$ and $y$ are substitutes and equations (20) and (21) hold. Now make the outputs 'better substitutes' by defining a new net benefit function $N^{\prime}(x, y)$ such that $N^{\prime}(\bar{x}, \bar{y})=N(\bar{x}, \bar{y})$ but for which $B_{x y}^{\prime}<B_{x y}$. Now at $(\bar{x}, \bar{y}),(21)$ no longer holds:

$$
\begin{equation*}
N^{\prime}(\bar{x}, \bar{y})<N^{\prime}(0, \bar{y}) \tag{29}
\end{equation*}
$$

That is, at $\bar{y}, \bar{x}$ makes a negative contribution to net benefit. Before $x$ and $y$ were made better substitutes, the maximum solution of (7a) described agency $x$ 's reaction to $\bar{y}$. But now $F^{x}(y)$ from (7a) describes an $x$ which brings (29) to equality; this must be an $x$ strictly less than $\bar{x}$. Meanwhile, equation (8a) describes an $x$ just barely less than $\bar{x}$, so that (8a) rather than (7a) now describes $H^{x}(\bar{y})$. Thus when $x$ and $y$ are made better substitutes, the old equilibrium is disturbed and agencies respond with offers characterized by (8a) for agency $x$ and (11a) for agency $y$ increasing the sponsor's net benefits.

## 4. More than two agencies

As the number of agencies increases beyond two, the constraints which apply to the sponsor's net benefit position in equilibrium increase exponentially. The sponsor may reject any single agency's offer, or the offers of any set of agencies. Each agency, additionally, brings its own equilibrium conditions to the problem. Rather than struggle anew with an extended list of inequalities, we will apply what has already been established to characterize the effect of creation of agencies on the sponsor's welfare.

Begin with an ( $\bar{x}, \bar{y}$ ) equilibrium and introduce agency $z$. The relation between $z$ and the two original outputs determines what happens to the sponsor's net benefits. It is both convenient and sensible to suppose that the sponsor has a 'memory', implying that possible positive output of a third agency cannot detract from the sponsor's net benefit.

If $x, y$, and $z$ are independent, sponsor's net benefit is unchanged at $(\bar{x}, \bar{y}, \bar{z})$. If $z$ is a substitute for either of $x$ or $y$ net benefits are increased. If $z$ is a substitute for $x$, for instance, regard $\bar{y}$ as fixed temporarily and the new equilibrium between $x$ and $z$ must increase net benefit over $(\bar{x}, \bar{y})$. (This is an application of proposition 3.) If $x, y$, and $z$ are complements, all bargaining pairs are described in Proposition 2 and there is no increase in net benefits. If $x$ and $y$ are substitutes but complementary to $z$, there may be an increase in net benefits. Positive output of $z$ may increase the substitutability of $x$ and $y$. (This depends on a third-order cross-partial $B_{x y z}$ ) disturbing the original equilibrium and leading to more offers which augment the sponsor's net benefits by $\epsilon$.

Generally, it is competition between agencies with substitutable outputs which generate net benefits to the sponsor. This competition can be induced
directly by introduction to substitute agencies, or indirectly through introduction of complementary agencies into situations where there are already agencies producing substitutes.

Not surprisingly, a 'competitive' supply structure is best for the sponsor. With free entry, no agency can produce positive output unless marginal benefit of its output is just equal to marginal cost (otherwise an entrant would offer a slightly lower output and induce the sponsor to reject the existing agency's output). With decreasing marginal cost this condition is fulfilled by one agency producing the efficient output; with constant costs, by any number of agencies producing together the efficient output; with increasing cost, by an infinite number of agencies producing together the efficient output. Note that with the behavioral assumptions of our model, two agencies producing the identical product with identical non-increasing costs are sufficient to lead to the most efficient output.

There is no doubt that we have stacked the deck in favor of a positive contribution of additional agencies if we assume the sponsor has a memory, and if, as we have also implicitly assumed, there are no fixed costs of an agency. That the sponsor has a memory is not unreasonable; that there are no agency fixed costs probably is unreasonable. Fixed costs could more than offset any benefits from increased competition induced by additional agencies.

## 5. Information to the sponsor

The sponsor possesses the overt authority to direct agencies' behavior. If the sponsor were fully informed about production possibilities, it could simply call for the outputs which maximize its welfare. The sponsor's overt authority is subverted, however, by its ignorance and the consequent need to take the 'word' of the agencies regarding cost. What agencies choose to reveal is determined by their own self-interest. We have modelled an extreme case of this in which agencies can make all-or-nothing offers to the sponsor. This shows how competition forces agencies to reveal more and sacrifice their own interests to the benefit of the sponsor.

Now suppose the sponsor can do hypothetical policy analysis and gain some information about cost independent of what agencies choose to reveal. The bargaining position of the agencies is clearly weakened, and the sponsor can only benefit from collection of (costless) information. In this section we want to stress a slightly less obvious point: information and competition are 'substitutes' in production of net benefits to the sponsor. That is, the increase in the sponsor's net benefit due to acquisition of a certain amount of information is inversely related to the degree of competitiveness of the agency supply structure. It should be clear from previous discussion that the presence of agencies with substitute products forces an agency to compete for the sponsor's funds by offering more net benefits from its output alone than it otherwise would. Armed with information the sponsor can direct an
agency to produce an output providing net benefits over and above those the agency was forced to offer by competition. The amount of benefits 'over and above', however, is less as the agency is forced by more competition to offer more net benefits. Outside information is less useful the more an agency is forced to reveal by competition. Any level of net benefits to the sponsor can be achieved by either giving the sponsor information or by increasing the competitiveness of the agency supply structure.

## 6. Conclusions

In the private sector of the economy, the structure of supply is taken to be one primary determinant of market performance. Changes in the structure are often viewed as policy options. By contrast, in the public sector, where the structure of supply is most directly under government control, introduction of rivalrous agencies is rarely considered as a way to improve performance. Agencies are of course simply creatures of governments, and it might generally be believed that the best way to achieve efficiency is simply to 'order' the optimal performance. This would be fine if analysts had enough information to know how the orders should read. But we are never so well informed. Various programs designed to produce centrally useful information, like cost-benefit analysis, can be helpful; but in the absence of a perfectly well-informed central administration, we ought to consider that agencies with their own goals may be able to influence the 'orders' they receive from the sponsor. To restate the central point of this paper: when a budget is a kind of bargain between agencies and the sponsor, the structure of the agency supply affects the sponsor's alternative uses for its funds, and thereby influences the terms of the budgeting bargains. The structure of supply is a policy variable with creation (destruction) of an agency decreasing (increasing) the bargaining power of agencies producing substitute products.

Creation of agencies producing substitute products, an unadulterated 'good' in our model, should not, however, proceed without caution. Agencies undoubtedly have fixed costs. What the U.S. needs is not another DOD or HEW! But at the same time elimination of apparently redundant agencies should also not proceed without caution. Imagine the horror among economists a proposal to eliminate 'redundant' private firms would evoke! There may be gains from consolidating The Bureau of Competition and The Antitrust Division, but as we have tried to point out, there may also be losses, more subtle, due to the more comfortable environment within which the new agency would be situated.

As our model has shown, agency competition for a sponsor's funds, even among agencies producing different products, can substantially benefit the ignorant sponsor. Models based on an agency or agencies supplying one product are unduly pessimistic in this regard. ${ }^{13}$ Our model provides theoretical support for the position that normative models of budgeting
based on decentralized decisions are legitimate rivals to the theory that efficient budgeting requires centralized information. ${ }^{14}$ Information and competitive structure are alternative and substitutable methods for achieving efficient allocations, just as they are in the private sector.

Creation of an inviolable domain of activity for an agency encourages wasteful expansion. Our model did not allow collusion. Collusion among agencies could be expected to take the form of an agreement to 'divide the market', that is, to mutually respect one another in sphere's of activity. This form of respect should be discouraged by the sponsor whenever possible. Thus, instead of assigning a new defense project to the 'appropriate' branch of the military, all services should be invited to propose how they would manage the project. There can be benefits to this even if all services are productively efficient.
'Super-agencies' may effect a degree of collusion among related agencies impossible for them to achieve independently. Centralization of all agencies dealing with energy for example may promise some savings in overhead but poses the dangers of an agency with unchallenged expertise and political clout. A monopoly replaces an oligopolistic structure of supply. ${ }^{15}$

## Glossary

(Values for $y$ are defined analogously.)

| $x$ | $=\quad$ output of agency $x$ |
| :--- | :--- |
| $B(x, y)$ | $=$ benefits of $(x, y)$ |
| $C^{x}(x)$ | $=$ cost of $x$ |
| $N(x, y)$ | $=$ net benefits of $(x, y)$ |
| $F^{x}(y)=$ | maximum $x$ sponsor would accept given acceptance of $y$ |
| $G^{x}(y)=$ | maximum $x$ sponsor would accept given rejection of $y$ |
| $H^{x}(y)=$ | maximum of $F^{x}(y)$ and $G^{x}(y) ;$ agency $x$ 's reaction function |
| $\bar{x}$ | $=$ equilibrium value of $x$ |
| $x^{*}$ | $=\quad$ value of $x$ which maximizes net benefits from agency $x$ alone |
| $\epsilon$ | $=\quad$amount by which an agency must increase the sponsor's net benefits <br> if the sponsor is to accept he offer and reject the offer of another |
|  | agency |

## Notes

1. 

The 538 count does not include the advisory committees ( 1,179 ), wholly-owned government corporations (18), mixed ownership government corporations (12), quasiofficial agencies (9), international organizations (93), or interagency committees (approx. 250). Reported by Robert G. Kaiser in the Washington Post, May 8, 1977, page C3.
2.

Statistical Abstract of the U.S., 1976, U.S. Department of Commerce, p. 273.
3.

Wildavsky (1964, p. 2).
4.

A brief literature review available from the authors places our model in the context of the public finance literature. Our model's most direct antecedent is Niskanen (1971).
5.

Wildavsky believes 'It is usually correct to assume that department officials are devoted to increasing their appropriations' (1964, p. 19). This advocacy role is considered by Congress and other budgetary participants as something 'natural' and 'inevitable' (1964, pp. 18-19, p. 164). Downs suggested that agencies are initially staffed by zealots' (1967, pp. 5-6). According to Downs, as the agency matures, zealots are replaced by bureaucrats who become advocates for reasons of power and prestige (1967, p. 17). He also discusses how internal structure of bureaus or agencies encourages advocacy (1967, p. 103). See also Lindblom (1965), (1968), Schultz (1968), Niskanen (1971). 6.
T.D. Schellhardt, 'Merger of Antitrust Division, FTC Unit is Ordered for Study by Attorney General,' Wall Street Journal, April 11, 1977, p. 4.
7.

Senator Proxmire complained about the lack of relevant information available to Congress:
"It is only slightly less than absurd that the Congress is expected to participate meaningfully in the policy-making process when it is not asked to consider alternatives, but only to approve or disapprove or to amend slightly at the margin' (1970, p. 421).

It would be wrong to think that the reality of frequent congressional cuts of agency proposals is inconsistent with the assumption that agencies know about how much to ask for. Congressmen and agencies each have their roles to play. Part of the congressman's role is to appear as guardian of the public purse. The cagey agency might know it can get $\$ 1,000,000$, but to do so requires a proposal of $\$ 1,200,000$. The agency takes a $\$ 200,000$ cut and keeps the congressional committee happy. A certain amount of padding and cutting is expected, but this adding on and taking away can be ignored in an analytical model concerned with outcomes.
8.

This is Niskanen's (1971) result. $\bar{x}$ is illustrated in Figure 1.
9.

This is equivalent to Hicks' definition (1961, p. 4).
10.

The assumption that $N(x, y)$ is convex upward limits the complementarity between $x$ and $y$. Without this assumption there could be diminishing marginal benefits but along some ray from the origin $N$ could increase indefinitely. The set $N(x, y) \geqslant 0$ would be unbounded and there would be no constraint on agency maximization.
11.

When $x$ and $y$ are substitutes isoquants are stretched NW to SE. See below. When $x$ and $y$ are independent there is no diagonal stretching although isoquants are generally not concentric circles.
12.

The isoquant for a $y$ in this range could have either of two shapes. If it is shaped like curve $i, H^{x}(y)$ is the maximum solution of (7a); if it is shaped like curve $j, H^{x}(y)$ is the maximum solution of (8a).

13.

For instance, Niskanen (1971).
14.

This is Lindlom's (1965) position.
15.

In his Newsweek column of May 23, 1977, titled 'A Department of Energy', Milton Friedman stresses the connection between consolidation and expansion of agencies using HEW as an example.

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