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# EXTERNALITIES IN ECONOMIES WITH IMPERFECT INFORMATION AND INCOMPLETE MARKETS\*

BRUCE C. GREENWALD AND JOSEPH E. STIGLITZ

This paper presents a simple, general framework for analyzing externalities in economies with incomplete markets and imperfect information. By identifying the pecuniary effects of these externalities that net out, the paper simplifies the problem of determining when tax interventions are Pareto improving. The approach indicates that such tax interventions almost always exist and that equilibria in situations of imperfect information are rarely constrained Pareto optima. It can also lead to simple tests, based on readily observable indicators of the efficacy of particular tax policies in situations involving adverse selection, signaling, moral hazard, incomplete contingent claims markets, and queue rationing equilibria.

Traditional discussions of externalities have emphasized the distinction between technological externalities, in which the action of one individual or firm directly affects the utility or profit of another, and pecuniary externalities, in which one individual's or firm's actions affect another only through effects on prices. While the presence of technological externalities imply, in general, that a competitive equilibrium may not be Pareto efficient, pecuniary externalities by themselves are not a source of inefficiency. The fact that prices change has, of course important consequences: there are both distributional and allocational effects. But, the distribution effects "net" out: gains for example, by firms whose prices increase—are precisely offset by losses—e.g., to individuals who must pay higher prices. And, there are no welfare losses from the allocation effects as long as the price changes involved are small: if firms are maximizing profits and individuals are maximizing utility, both facing prices that correctly reflect opportunity costs, then standard envelope theorem arguments imply that changes in profits or utility induced by changes in allocations (resulting from any small change in prices) are negligible.

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At the same time, pecuniary externalities have significant welfare consequences when there are distortions in the economy (e.g., from monopolies, technological externalities, or distorting taxes). An important determinant of the optimal tax on one commodity is for instance a calculation of its indirect effect on government revenue raised from other taxes.<sup>1</sup> It has not, however, been widely recognized that the distortions that arise in economies in which there is imperfect information and incomplete markets—for practical purposes, all economies—result in there being real welfare consequences of what would otherwise be viewed as purely pecuniary effects. As a result, economies in which there are incomplete markets and imperfect information are not, in general, constrained Pareto efficient. There exist government interventions (e.g., taxes and subsidies) that can make everyone better off. Moreover, the distortions that arise from imperfect information or incomplete markets often look analytically like externalities of the familiar technological sort, and viewing them in this way helps identify the welfare consequences of government interventions.

With these observations in mind, the objective of this paper is to develop a general methodology both for analyzing the impact of externalities and for calculating optimal corrective taxes in a general equilibrium context. The approach developed can be applied easily not only to conventional technological externalities but to the more subtle class of externalities associated with imperfect information and incomplete markets. We show how, in many cases, not only can it be demonstrated that there exist Pareto-improving government interventions, but also that the kind of intervention required can be simply related to certain parameters that, in principle, are observable.

The paper is divided into four parts. The first presents the model used and develops the general methodology. Section II applies this methodology to a number of widely discussed welfare problems involving imperfect information and incomplete markets. Section III discusses some other important applications and extensions of the analysis. Finally, Section IV is a brief conclusion.

1. The importance of these indirect effects was emphasized, for instance, by Harberger [1971] in his classic paper.

## I. THE BASIC MODEL AND RESULTS

The agents in the model consist of households, firms, and a government with the following characteristics.

## A. Households

Households maximize a utility function,

$$u^h(x^h, z^h), \quad h = 1, \dots, H,$$

where

$x^h = (x_1^h, \bar{x}^h) \equiv$  consumption vector of household  $h$ ,  $x_1^h$   
is consumption of the numeraire good,  $\bar{x}^h$   
 $= (x_2^h, \dots, x_N^h)$  is consumption of the  
 $N - 1$  nonnumeraire goods,  
 $z^h \equiv$  vector of  $N^h$  other variables that affect  
the utility of household  $h$  (e.g., levels of  
pollution, average quality of a good  
consumed).

Households maximize  $u^h$  subject to a budget constraint of the form,

$$x_1^h + q \cdot \bar{x}^h \leq I^h + \sum_F a^{hf} \cdot \pi^f,$$

taking  $q$ ,  $\pi^f$ ,  $I^h$ ,  $a^{hf}$ , and  $z^h$  as fixed, where

$q \equiv$  a vector of prices of the  $N - 1$  nonnumeraire goods,

$\pi^f \equiv$  profits of firm  $f$ ,

$a^{hf} \equiv$  fractional holding of household  $h$  in firm  $f$ ,

$$\sum_H a^{hf} = 1,$$

$I^h \equiv$  a lump sum government transfer to household  $h$ ,

$$I \equiv (I^1, \dots, I^H).$$

We shall also use

$E^h(q, z^h, u^h) \equiv$  the expenditure function of household  $h$  that gives the minimum expenditure necessary to obtain a level of utility  $u^h$ , when prices are  $q$  and  $z^h$  is the level of "other" variables.

It is well-known that

$$\hat{x}_k^h(q; z^h, u^h) \equiv \text{the compensated demand for good } k \text{ given } z^h \text{ and } u^h \text{ fixed}$$

(where the caret is used to distinguish compensated from uncompensated demand functions):

$$(1) \quad \equiv \left. \frac{\partial E^h}{\partial q} \right|_{z^h, u^h}.$$

Finally,

$$x^h(q, I, z^h) \equiv (x_1^h(q, I, z^h), \bar{x}^h(q, I, z^h)) \equiv \text{the demand function (uncompensated) of household } h.^2 \text{ We shall assume that this function is differentiable.}^3$$

### B. Firms

Firms maximize the profit function,

$$\pi^f = y_1^f + p \cdot \bar{y}^f,$$

where

$$y^f = (y_1^f, \bar{y}^f) \equiv \text{production vector}^4 \text{ of firm } f \text{ with } y_1^f \text{ and } \bar{y}^f \text{ defined analogously to } x_1^h \text{ and } \bar{x}^h,$$

$$p \quad \equiv \text{vector of producers' prices for the } N - 1 \text{ nonnumeraire goods.}$$

Firms maximize profits subject to the constraint that,

$$y_1^f - G^f(\bar{y}^f, z^f) \leq 0,$$

where

$$G^f \equiv \text{a production function of the usual sort,}$$

2. The household demand function depends on the entire vector of transfers since both  $z^h$  and  $z^1, \dots, z^F$  (which determine  $\pi^1, \dots, \pi^F$  and hence household income) may depend on the consumption choices of other households. In a pure exchange economy,  $x^h(q, I^h; z^h(q, I)) = x^h(q, I)$ . Also for the sake of expositional simplicity, household factor endowments have been arbitrarily set to zero. This has no substantive impact on the analysis.

3. The problem of justifying this kind of differentiability assumption is examined in detail by Starrett [1980], who makes a similar assumption in a slightly different context. The difficulty here is that the usual convexity assumptions of preferences and production functions will not guarantee differentiability. The external effects may create discontinuities. The "excess demand" functions used here include the effect of prices on quantities both directly and indirectly via their impact on externality-generating activities (i.e., through their impact of  $z^f$  and  $z^h$ ) which, in turn, affect consumption and production choices.

4.  $y_k^f < 0$  represents an input.

$z^f \equiv$  vector of other  $N^f$  variables affecting firm  $f$   
analogously defined to  $z^h$ .

The firm's maximum profit function,

$$\pi_*^f(p, z^f),$$

has the property that

$$(2) \quad \left. \frac{\partial \pi_*^f}{\partial p_{k1}} \right|_{z^f} = y_k^f, \quad k = 1, \dots, N,$$

where  $y_k^f$  here denotes the profit-maximizing level of the production variable in question. Finally,

$$y^f(p, z^f) \equiv (y_1^f(p, z^f), \bar{y}^f(p, z^f)) \equiv \text{supply function of firm } f.$$

We shall assume that this function, like the demand function, is differentiable.

### C. Government

The government produces nothing, collects taxes, distributes the proceeds, and receives a net income,

$$R \equiv t \cdot \bar{x} - \sum_H I^h,$$

where the tax  $t$  is just the difference between consumer and producer prices,

$$t \equiv (q - p),$$

and

$$\bar{x} \equiv \sum_H \bar{x}^h \text{ (i.e., the sum of nonnumeraire consumption).}$$

### D. Equilibrium and Efficiency

An initial equilibrium with no taxes and  $I^h = 0$  for all  $h$ , will be assumed to exist.<sup>5</sup> At this equilibrium,  $p = q$ , and<sup>6</sup>

5. As described so far, the model may not, of course, have an equilibrium price vector. However, having noted that possibility, it is still worth investigating the welfare implications of any equilibria that may exist. The case for this is made fully and compellingly by Starrett [1980]. We shall also ignore the problem of free goods. Accounting for them would merely complicate the analysis without altering any basic results.

6. At the most general level,

$$(3a) \quad z^h = z^h(x^1, \dots, x^h, y^1, \dots, y^f).$$

We must solve simultaneously (3) and (3a) for the endogenous variables  $\{x^h, z^h, x^f, z^f\}$  in terms of the exogenous variables  $\{t, I\}$ .

$$(3) \quad \bar{x}(q, I, z) - \sum_F \bar{y}^f(p, z) = 0.$$

A simple test of the Pareto optimality of this equilibrium is to ask whether there exists a set of taxes, subsidies, and lump sum transfers that would (a) leave household utilities unchanged and (b) increase government revenues (assumed to be consumed in the numeraire good). This, in turn, implies that, if the original equilibrium is Pareto optimal, the problem,

$$(4) \quad \max_{t, I} R \equiv t \cdot \bar{x} - \sum_h I^h,$$

subject to

$$(5) \quad I^h + \sum a^{hf} \pi^f = E^h(q, z^h, \bar{u}^h),$$

where  $\bar{u}^h$  = competitive equilibrium utility levels, and  $z^h, z^f, \pi^f, p$ , and  $q$  are functions of  $t$  and  $I$ , has a solution at  $t = 0$ .

This is, of course, a *necessary but not sufficient* condition for (constrained) Pareto optimality. Clearly, if we can find a set of tax-subsidy interventions that can make everyone better off, the economy is not Pareto efficient. But there might exist other forms of intervention, such as quotas, that might generate Pareto improvements even when no simple tax-subsidy scheme could do so.<sup>7</sup> To see when the solution to (4) entails  $t = 0$ , note that, along the constraint of equation (5),

$$(6) \quad \frac{dI^h}{dt} + \sum_F a^{hf} \left( \pi_z^f \frac{dz^f}{dt} + \pi_p^f \frac{dp}{dt} \right) = E_q^h \frac{dq}{dt} + E_z^h \frac{dz^h}{dt},$$

where

$$\frac{dI^h}{dt} \equiv \text{change in lump sum income per unit change in tax}^8 \\ \text{required to keep the individual at the given level of utility,}$$

7. At the same time, it might be noted that we ignore any discussion of the political processes by which the tax-subsidy schemes described below might be effected. Critics may claim that as a result we have not really shown that a Pareto improvement is actually possible.

8. All the derivatives of  $z$  (and  $p$  and  $q$ ) with respect to  $t$  should be viewed as total derivatives, taking into account the associated changes in  $I_1, \dots, I_H$  (which, in principle, may affect  $z$ ) as well as the direct effect of  $t$ .

$$\pi_z^f \equiv \left[ \frac{\partial \pi_*^f}{\partial z^f} \right] = \frac{\partial G^f}{\partial z^f}, \text{ an } N^f \text{ element vector,}$$

$$E_z^h \equiv \left[ \frac{\partial E^h}{\partial z^h} \right] = \left[ \frac{\partial u^h}{\partial z^h} \right] \text{ (with } u^h \text{ suitably normalized), an}$$

$N^h$  element vector.

But,  $dq/dt = I_{N-1} + dp/dt$  (here  $I_{N-1}$  is an identity matrix). Therefore, substitution into (5) and rearrangement of terms yields

$$(7) \quad E_q^h + (E_q^h - \sum_f \alpha^{hf} \pi_p^f) \frac{dp}{dt} = \frac{dI^h}{dt} + \left\{ \sum_F \alpha^{hf} \pi_z^f \frac{dz^f}{dt} - E_z^h \frac{dz^h}{dt} \right\}.$$

The left-hand side of (7) is the traditional pecuniary (or redistributive) effect of the tax, while the bracketed term on the right-hand side is the externality effect. So far, the derivation of equation (7) involves nothing more than keeping track of the impact on household  $h$  of a small change in taxes  $dt$ , where this impact includes the effects of any associated equilibrium price changes. Substitution of equations (1) and (2), summation over all households, and use of the fact that  $\sum_h \alpha^{hf} = 1$  help to simplify the distributive impact of the initial tax change. Thus,

$$\bar{x} + (\bar{x} - \bar{y}) \frac{dp}{dt} = \sum_H \frac{dI^h}{dt} + \left( \sum_F \pi_z^f \frac{dz^f}{dt} - \sum_H E_z^h \frac{dz^h}{dt} \right).$$

It may be helpful here to recall how (1) and (2) are derived: an envelope theorem is used to eliminate the allocative effects of the tax-induced price changes. This is why no terms appear directly reflecting these allocative effects. Next,  $\bar{y} \equiv \sum_F \bar{y}^f = \bar{x}$  in any market equilibrium. Therefore, the distributive effects, i.e.,  $(\bar{x} - \bar{y})dp/dt$ , "net" out. And the total compensating payments that the government must make to satisfy the constraint (5) amount to

$$(8) \quad \sum_H \frac{dI^h}{dt} = \bar{x} - \left( \sum_F \pi_z^f \frac{dz^f}{dt} - \sum_H E_z^h \frac{dz^h}{dt} \right).$$

Now differentiating the objective function (4) with respect to  $t$ , we obtain

$$(9) \quad \frac{dR}{dt} = \bar{x} + \frac{d\bar{x}}{dt} \cdot t - \sum_H \frac{dI^h}{dt}.$$

Substitution from (8) into (9) yields



$$(10) \quad \frac{dR}{dt} = \frac{d\bar{x}}{dt} \cdot t + (\Pi^t - B^t),$$

where

$$(11) \quad \Pi^t \equiv \sum_F \pi_z^f \frac{dz^f}{dt},$$

$$(12) \quad B^t \equiv \sum_H E_z^h \frac{dz^h}{dt},$$

which is the derivative of  $R$  along directions in which the compensation constraint is satisfied. This can be used as a measure of the net change in welfare. The disappearance of the  $\bar{x}$  term here is due to the elimination of one final distributive effect that is particular to the tax change. The total compensation to households is offset in part by the increase in tax revenue to the government embodied in the term  $\bar{x} \cdot dt$ . The remaining terms in equation (10) summarize the "pecuniary" effects of the tax change that cannot be ignored. These depend on existing distortion whether in the form of taxes (i.e.,  $d\bar{x}/dt \cdot t$ ) or technological externalities (i.e.,  $\Pi^t$  and  $B^t$ ).

For the initial equilibrium to be Pareto optimal,  $dR/dt$  must equal zero at  $t = 0$ , which implies that

$$(13) \quad \frac{dR}{dt} = (\Pi^t - B^t) = 0.$$

Thus, Pareto optimality depends on the absence of any  $z$ 's that change with taxes and affect either profits or household utilities.<sup>9</sup>

The defining characteristics of externalities, which (in traditional language) are "nonpecuniary" and, therefore, justify some form of government intervention, is that they enter utility or profit functions in the form of the  $z$ -variables. The variables involved may, of course, be determined by the market interactions of agents (e.g., average product qualities, search times, average levels of unobservable effort or, with incomplete markets, future prices) and this will be the case in the examples analyzed below.

9. If the economy were Pareto optimal,  $dR/dt$  would equal zero, so we need not concern ourselves with how the government disposes of any excess revenues. For the same reason, (14) below characterizes the optimal tax structure for any rule for the disposition of net government revenues. (The simplest rule is for the government to spend all of its excess revenue on the numeraire good, in which case (3) is always satisfied in equilibrium.)

Except in the special case (which is unlikely to hold generically) where  $\Pi^t$  and  $B^t$  exactly cancel each other out, the existence of these externalities will make the initial equilibrium inefficient and guarantee the existence of welfare-improving tax measures.

We should review here the important assumptions that underlie our analysis: (a) firms are competitive profit maximizers and individuals are competitive utility maximizers; this allows us to use the envelope theorem, to say that there is no welfare effect from the changes in actions induced by the changes in prices; (b) demand equals supply (and all profits accrue to individuals within the economy); this allows us to cancel out the distributive or *transfer effects*, the gains from price increases to sellers (owners of firms who are producers) being just offset by the losses to buyers.

### E. Optimal Taxes

Equation (10) not only allows us to ascertain whether an economy is a constrained Pareto optimum, but also provides a simple set of necessary conditions characterizing the optimal level of taxes in the presence of externalities. Since  $dR/dt = 0$  is necessary for optimality, optimal tax levels have the property that

$$(14) \quad t \cdot \frac{d\bar{x}}{dt} = -(\Pi^t - B^t)$$

or

$$(14') \quad t = -(\Pi^t - B^t) \left( \frac{d\bar{x}}{dt} \right)^{-1}.$$

The left-hand side of (14) is the marginal deadweight loss from the distortion in consumption associated with an increase in the tax.<sup>10</sup> The right-hand side is the gain from reduction in the externalities. At the optimum, the marginal gain from the reduction

10. Heuristically, the marginal deadweight loss from an increase in the tax is just the difference between the increased income that would have to be given to an individual to keep him at the same level of utility and the extra revenue received by the government. In the simple case where producers' prices are fixed,

$$\begin{aligned} \frac{d(DWL)}{dt_i} &= \sum_H \frac{dE^H(q, \bar{u})}{dq_i} - \frac{d(t \cdot \bar{x})}{dt_i} \\ &= x_i - x_i + t \cdot \frac{d\bar{x}}{dt_i}. \end{aligned}$$

in the externality should just equal the marginal deadweight loss from the (direct effect of the) tax.

A simple example may help clarify the implications of (14). Assume that a tax on alcohol reduces automobile accidents, and that individuals, in deciding on the level of care, do not fully take into account the social costs of their actions (e.g., because they are partially insured). Then a tax on alcohol will always be initially beneficial. However, successive tax increases will increase the deadweight loss: the marginal value of alcohol consumption to the individual will exceed (by increasing amounts) the producer cost. The tax should be increased until the marginal deadweight loss (the constant rate loss in tax revenue) exactly balances the marginal benefits of reductions in the accident costs that have not been internalized by the individual (the accident externalities).<sup>11</sup>

## II. APPLICATIONS

The remainder of this paper is devoted to applying equation (13) to a variety of familiar situations, to ascertain conditions under which a small tax or subsidy will be welfare enhancing. One of the main virtues of our methodology is the ease with which it can be applied, in particular, to situations where information is imperfect and markets incomplete, to show that in such a situation there virtually always exists a tax subsidy that is Pareto improving. But before applying our methodology to these somewhat unfamiliar situations, it may be useful to see how it works in the more familiar context of some pre-existing (assumed to be fixed tax) distortions.

### A. Tax Distortions

For simplicity, we assume that there exists a single tax distortion, say on commodity 1, generating revenue  $t_1x_1$ , the proceeds of which are redistributed back to households according to

11. The left-hand side is sometimes referred to as the "constant" rate loss in tax revenue, where constant rate changes in tax revenue are the changes in revenue that would have occurred at the existing tax rates.

Two further points about this optimal tax formula in the presence of externalities are worth making. First, because the impacts of  $t_i$  and  $t_j$  on externality distortions will not, in general, be equal, the standard equiproportionate reduction results do not obtain. Second, we have assumed that the government can adjust the  $I^h$  lump sum transfers to offset any distributional effects. If it cannot, and we ask what tax structure maximizes social welfare, then the formulae corresponding to (14) will employ distributional weights. See Atkinson and Stiglitz [1980].

a fixed formula; i.e., the  $h$ th household gets a share  $\beta^h$  of the tax revenue from the first commodity. We then take the somewhat unnatural step in this case of rewriting the tax distortion as a traditional technological externality, defining

$$z^h = \beta^h t_1 x_1, \quad \sum_H \beta^h = 1,$$

since tax proceed distributions are "externalities" to each household. Clearly now, the individual's utility (and his demands) are functions not only of all prices, but also of  $z^h$ . Directly applying (13), we obtain

$$\left. \frac{\partial R}{\partial t_i} \right|_{t_i=0} = t_1 \sum_H \beta^h \left( \frac{dx_1}{dt_i} \right)_{\bar{u}} = t_1 \left( \frac{dx_1}{dt_i} \right)_{\bar{u}} \geq 0 \quad \text{as} \quad \left( \frac{dx_1}{dt_i} \right)_{\bar{u}} \geq 0.$$

A small tax (subsidy) on any commodity that is a Hicks substitute (complement) to the first commodity is welfare enhancing.

### B. Adverse Selection

The simplest imperfect information case in which the analysis can be applied is to markets with asymmetrically distributed information and heterogeneous quality.<sup>12</sup> We shall assume that there is only a single commodity about which purchasers are uninformed and that there are no other externalities (or other distortions). Sellers know the quality of what they are selling. Buyers know only the average quality in the market as a whole. Buyers will be assumed to draw randomly from the market in which the commodity in question is offered for sale. We shall assume, in addition, that buyers are perfectly informed about and care only about the *average* quality of what they buy.<sup>13</sup> (Realistically, buyers may also care about the range of possible qualities, but taking this into account would change the analysis only in obvious ways and would greatly increase its complexity.)<sup>14</sup> The situation corresponding perhaps most closely to this simple model

12. The basic model for these situations was developed by Akerlof [1970].

13. As Stiglitz [1975a] noted earlier, ignorance (imperfect information) acts like a tax/subsidy, increasing the wage received by an individual above his marginal product for low-productivity workers, and decreasing it for high-productivity workers.

14. This simple model applies equally well to a situation in which buyers purchase only a limited number of items and care about the individual qualities of each. In that case *ex ante* expected utility (the appropriate welfare measure) will depend on the mean and spread of the distribution of "quality" in the market pool.

is a labor market in which firms hire blindly from a pool of workers of heterogeneous quality.<sup>15</sup>

We let  $\theta$  denote the quality of each unit of the heterogeneous commodity, and  $\bar{\theta}$  denote the average quality in the marketplace. In terms of the model of this paper, the  $z^h$  (externality) vectors will consist of a single element that is equal to  $\bar{\theta}$  (although households that do not purchase the commodity may have  $du^h/dz^h = 0$ ). Similarly,  $z^f$  for all firms will have a single element equal to  $\bar{\theta}$ . Formally,

$$E^h = E^h(q; \bar{\theta}),$$

and

$$\pi^f = \pi^f(p; \bar{\theta}).$$

Under these circumstances, equation (13) for a small tax  $dt$  becomes

$$(15) \quad \frac{dR}{dt} = \left[ \sum_F \pi^f - \sum_H E^h \right] \cdot \frac{d\bar{\theta}}{dt}.$$

Since  $\pi^f$  increases and  $E^h$  decreases with  $\bar{\theta}$ , this means that any intervention which increases average quality in the marketplace is beneficial. Thus, *any small tax that increases the quality of the heterogeneous commodity is always beneficial.*

What is surprising about this result is its simplicity. The fact that an increase in  $\theta$  involves the sale of higher quality inputs by some households suggests the need for a careful balancing of the increased cost of these sales by owner households against the benefits to purchasers. Yet no such calculation is implied by equation (15). The necessary balancing of the costs and benefits of selling higher quality items is being done by owner households

15. A question that might arise is whether agents, observing the dependence of quality on price, will behave in the manner described here. We assume here (following Akerlof) that the uninformed agents do not act strategically. This assumption seems reasonable, for instances when labor is engaged at a union hall, in which there are a large number of employers. Then the supply of laborers will essentially be unaffected by any single firm. Hence, a firm will have no incentive to pay a wage in excess of the market wage, and cannot obtain any workers at a lower wage. But there are other circumstances in which a single purchaser can obtain information about the characteristics of the particular good the seller is trying to sell by a variety of devices. See, e.g., Stiglitz [1976] and Stiglitz and Weiss [1981].

in the process of maximizing utility. This accounts for the simple form of the final policy prescription.

A typical example of tax changes leading to changes in average ability arises where different ability groups have different labor supply elasticities. If higher ability workers have greater supply elasticities than do low ability workers, a small proportionate wage subsidy will increase average quality.

Finally, it should be noted that there is, at least in principle, an observable basis for judging the effectiveness of government tax policy. Assuming that the *average* "quality" of labor entering a particular market can be monitored (short of determining the "quality" of each individual worker) by, for example, taking a statistical sample, any policy of "small" taxes that increases this quality is a beneficial one.

A question that naturally arises at this juncture is whether the compensations required by (5) can actually be carried out given the information available to the government. The answer depends, not unnaturally, on what the government knows and the extent to which lump sum taxes are available. If the government is restricted to commodity taxes and a uniform lump sum tax and knows the characteristics of each of the  $M$  classes of consumers (but not the class to which any particular individual belongs) then Pareto-improving commodity taxes will, in general, exist as long as the number of taxable commodities strictly exceeds  $M$  (i.e.,  $N > M$ ). Let the government restrict itself to tax changes that keep each class of consumers, except the first, at a given level of utility. As a rule, this will require  $M - 1$  taxes (one for each group except the first). Then let the government change the tax on a further commodity making simultaneous changes in the  $M - 1$  other taxes to keep the classes of consumers at all their given levels of utility. If the original equilibrium is not a Pareto optimum, then, in general, a composite tax change of this kind will exist that raises revenue.<sup>16</sup>

16. In the subsequent analysis we shall ignore these issues. The questions are, however, of central importance: the failure to take account of what information is at the disposal of the government provides one of the most telling criticisms, both of the standard compensation criteria as well as the New Welfare Economics, which assumed that all lump sum transfers were feasible. The New Welfare Economics and the Theory of Pareto Efficient Taxation [Stiglitz, 1982a, 1985] focus explicitly on these issues. The empirical information required of the government to implement Pareto improvements is, of course, much greater when compensations must be done through the commodity tax system.

### C. Signaling-Screening

The previous section considered situations where there was no signal that a seller with a higher quality commodity (a more productive worker) could use to distinguish himself from lower quality workers. In many cases, such signals, like education, can be obtained, but at a cost. Though there has been considerable work describing the resulting equilibrium (and analyzing the conditions under which an equilibrium exists)<sup>17</sup> the welfare properties of these equilibria have received surprisingly little attention. This is perhaps because of the result, noted in Rothschild-Stiglitz, that the competitive equilibrium, when it exists, has the property that it maximizes the welfare of the better-off individual, subject to the self-selection constraint. This suggests, in turn, that if the government has no more information available to it than private firms (and thus in redistributing income, must rely on the same self-selection constraints) it cannot make a Pareto improvement. This conclusion, however, is wrong. Taxes on goods or wages, which firms and individuals take as given, may change the extent of signaling, the average quality of those obtaining each signal and the wages paid to each category of signaling workers. Many of the resulting transfer and allocation effects will indeed disappear from a calculation of the consequent change in welfare. However, the average qualities of each signaling group are externalities just as average quality is in the adverse selection case. There remain, therefore, direct effects of any quality changes on purchasers, and these it can be shown will not in general net out: signaling market equilibria are essentially never constrained Pareto efficient.<sup>18</sup>

We develop here a simplified version of the signaling model, in which there is a single signal, which can be purchased at a cost; those who purchase the signal have mean quality  $\bar{\theta}_1$ , those who do not have mean quality,  $\bar{\theta}_2$ .<sup>19</sup> Since signals are costly and

17. See Spence [1973], Rothschild and Stiglitz [1976, and Wilson [1977] among others.

18. Earlier analyses [Stiglitz, 1975a] showed that there might exist multiple equilibria, some of which Pareto dominated others. The analysis here, however, shows that in general, each of the equilibria themselves can be improved upon with a simple set of taxes.

19. The version of the model presented here is considerably simpler than the standard formulation, where there are as many different signals (education levels) as there are types of individuals, and in which therefore there is an entire sequence of self-selection constraints. It is possible to apply the approach of this paper to equilibria of this sort. Externalities arise because the actions of one firm or individual affect the self-selection constraints of others. The essential insights are conveyed by the formulation presented. See Greenwald and Stiglitz [1985].

wages must, therefore, depend positively on signals, we shall assume that  $\bar{\theta}_1 > \bar{\theta}_2$ . For simplicity, we assume that only firms buy labor.

From application of equation (13), the net impact of a small tax  $dt$  is

$$(16) \quad \frac{dR}{dt} = \sum_i \sum_F \frac{\partial \pi^f}{\partial \theta_i} \cdot \frac{d\bar{\theta}_i^f}{dt}.$$

If we assume that firms draw at random from the pools of workers with and without signals and that each firm hires a large number of workers, we can rewrite (16) as

$$(17) \quad \frac{dR}{dt} = \sum_i \frac{\partial \bar{\theta}_i}{\partial t} \left[ \sum_F \frac{\partial \pi^f}{\partial \theta_i} \right].$$

Since  $\partial \pi^f / \partial \bar{\theta}$  is positive (i.e., higher average worker quality leads to higher profits), it follows immediately that any tax which increases the average quality in both the signaling and nonsignaling pools is beneficial. This would be true of a tax that discouraged workers who are below the average of those in the signaling pool but above the average of those in the nonsignaling pools, from acquiring the signal. Again the simplicity of this result follows from the fact that the many complicated "pecuniary" transfer effects and the effects of quality on a firm's hiring decisions can be ignored. We now make several simplifying assumptions to sign the right-hand side of (17).

Assume that the value of higher quality to a firm is directly proportional to the number of workers of a particular type that it hires; for instance, if the production process is separable, so total output  $y_0^f$  is the sum of the outputs of each individual;<sup>20</sup> i.e.,

$$y_0^f = \sum_i n_i^f y_{0i}(\hat{y}_i^f, \bar{\theta}_i),$$

where  $n_i^f$  is the number of workers of type  $i$  hired by firm  $f$ , and  $y_{0i}$  is the output of a worker of type  $i$  (given inputs per worker of  $\hat{y}_i^f$ ). Then

$$\sum_F \frac{d\pi^f}{\partial \theta_i} = n_i \sum_F \left[ \frac{n_i^f}{n_i} \right] \left[ \frac{\partial y_{0i}^f}{\partial \theta_i} \right] \equiv n_i \frac{\partial \bar{y}_0}{\partial \theta_i}, \quad \bar{y}_0 = \sum \frac{n_i^f}{n_i} y_{0i},$$

20. The results stated below only require that the marginal effect of an improvement in quality be proportional to the number of workers.



where  $n_i$  is the total number of workers of type  $i$ . Thus,

$$(18) \quad \frac{dR}{dt} = n_1 \frac{\partial \bar{\theta}_1}{\partial t} \left[ \frac{\partial \bar{y}_{01}}{\partial \bar{\theta}_1} \right] + n_2 \frac{\partial \bar{\theta}_2}{\partial t} \left[ \frac{\partial \bar{y}_{02}}{\partial \bar{\theta}_2} \right].$$

If we further assume that the overall average quality of the labor force is unaffected by the signal and is fixed (i.e.,  $n_1\bar{\theta}_1 + n_2\bar{\theta}_2$  is fixed), then

$$n_2 \frac{\partial \bar{\theta}_2}{\partial t} + n_1 \frac{\partial \bar{\theta}_1}{\partial t} + \frac{\partial n_1}{\partial t} (\bar{\theta}_1 - \bar{\theta}_2) = 0.$$

Substitution from this expression into (18) yields

$$(19) \quad \frac{dR}{dt} = \left[ n_1 \frac{\partial \bar{\theta}_1}{\partial t} \right] \left[ \frac{\partial \bar{y}_{01}}{\partial \bar{\theta}_1} - \frac{\partial \bar{y}_{02}}{\partial \bar{\theta}_2} \right] - \frac{\partial n_1}{\partial t} \left[ \frac{\partial \bar{y}_{02}}{\partial \bar{\theta}_2} \right] (\bar{\theta}_1 - \bar{\theta}_2).$$

The first term in (19) captures the "sorting" value of the signal. It is the improvement in quality in the signaling pool (i.e.,  $\partial \bar{\theta}_1 / \partial t$ ) multiplied by the differential value of "quality" for workers from the signaling compared with the nonsignaling pool. If "quality" is more important for signaling workers, then this term will be positive, and therefore a tax that increases the quality of the signaling pool will tend to be beneficial. If this increase in quality is achieved by reducing the number of workers who signal (i.e.,  $\partial n_1 / \partial t < 0$ ), then the second term in (19) will also be positive (since  $\bar{\theta}_1 - \bar{\theta}_2 > 0$  and  $\partial \bar{y}_{02} / \partial \bar{\theta}_2 > 0$ ), and the tax will be unambiguously beneficial (remember that this applies to the case where overall average quality is constant).

Furthermore, if there is no "sorting" effect (pure hierarchical screening) (i.e.,  $\partial \bar{y}_{01} / \partial \bar{\theta}_1 = \partial \bar{y}_{02} / \partial \bar{\theta}_2$ ), then

$$(20) \quad \frac{dR}{dt} = - \frac{\partial n_1}{\partial t} \left[ \frac{\partial \bar{y}_{02}}{\partial \bar{\theta}_2} \right] (\bar{\theta}_1 - \bar{\theta}_2).$$

and a small tax that reduces the amount of signaling is beneficial.

Finally, if the original equilibrium involves no signaling (i.e.,  $n_1 = 0$ ), then (20) again applies (with the reservation that  $\partial n_1 / dt$  now refers to the right-hand derivative at  $n_1 = 0$ ).

#### *D. Moral Hazard*

It has long been recognized that the provision of insurance attenuates incentives for accident avoidance. The insurance company knows this and takes this into account in designing the insurance contract that frequently has coinsurance and deduct-

ibility provisions. There is a tradeoff between the deadweight loss from the failure (with insurance) to take adequate accident avoidance precautions, and the welfare loss from risk bearing. But there is a widespread presumption that in competitive markets, the tradeoff is done in an efficient manner: indeed, the competitive equilibrium contract is generally described as that contract which maximizes individuals' expected utility, subject to the insurance company at least breaking even.<sup>21</sup> Hence, there has been a presumption that competitive economies, even with moral hazard, are constrained Pareto efficient. (Clearly, welfare would be higher if information were costless, so the insurance company could monitor the actions of the insured, in which case, the provision of the insurance would be contingent upon the individual taking certain accident prevention actions. But this is an irrelevant comparison.) This presumption is, unfortunately, wrong, and our framework provides an easy way of seeing this. The simplest way of doing so entails effectively embedding the zero-profit constraint on the insurance company into the utility function,<sup>22</sup> then the price an individual pays for insurance depends on the average level of accident avoidance of those who purchase insurance, which represents an externality to an individual purchaser. The government, by subsidizing complements of accident avoidance activities and taxing substitutes, encourages accident avoidance, reduces the externality, and improves welfare.

Assume for simplicity that the universe of insured agents consists of identical households and that a scalar level of effort that reduces the expected loss from accidents cannot be observed by insurers. Let households maximize,<sup>23</sup>

$$\mathcal{E}[U^h(x^h, \mu^h, e^h)],$$

subject to the constraint that

$$q \cdot (x^h - w^h) + \gamma(\mu^h, \bar{e}) - I^h - \sum_F a^{hf} \pi^f \leq 0,$$

21. For a discussion of the nature of market equilibrium with moral hazard, see Pauly [1974], Shavell [1979], and Arnott and Stiglitz [1983].

22. This is not the only way of approaching the problem with our framework, but it provides the results most directly. We could, alternatively, treat purchasers of insurance as a heterogeneous pool (similar to adverse selection) with an average quality, which in this instance would be the level of care exercised in avoiding accidents.

23. Accident losses are subsumed in this function. This formulation assumes that the individual commits himself to *all* nonnumeraire expenditures prior to knowing whether there will be an accident. Our results hold for more general formulations.

where  $\xi$  denotes an expectation across states of nature,  $\mu^h$  is a vector of insurance payments across states of nature (i.e.,  $\mu_1^h$ , the first element of  $\mu^h$ , is the insurance payment made to household  $h$  in state of nature 1),  $\gamma(\mu^h, e)$  is the premium paid for insurance,  $e^h$  is the level of "care" exercised by household  $h$ , and  $\bar{e}$  is the "average" level of care exercised by all households; i.e.,

$$\bar{e} = \frac{1}{H} \sum_H e^h$$

and  $w^h$  is the individual endowment vector.

With constant returns to scale in the insurance industry and risk-neutral investors, equilibrium in the insurance industry implies that

$$\gamma(\mu^h, \bar{e}) = \ell'(\mu^h | \bar{e}).$$

This can be substituted into the household budget constraint so that (the competitive equilibrium is as if) households choose  $e^h$ ,  $x^h$ , and  $\mu^h$  in order to maximize  $\ell' [U^{*h}]$  subject to the constraint,<sup>24</sup>

$$q \cdot (x^h - w^h) + \ell'(\mu^h | \bar{e}) - I^h - \sum_F a^{hf} \pi^f \leq 0,$$

where the function  $U^* \equiv \ell' [U^h(x^h, \mu^h, e^h)]$  can be treated as a normal utility function. We derive an expenditure function, as before,

$$E^h = E^h(q, U^*, \mu^h, \gamma(\mu^h, \bar{e}))$$

where, for the moment, we take  $\mu^h$  as given and  $\bar{e}$  is our "z" (externality) variable. Application of equation (13) implies that the net impact per unit of tax  $dt$  is<sup>25</sup>

$$(21) \quad \frac{dR}{dt} = \sum_H \frac{d \ell'(\mu^h | \bar{e})}{d \bar{e}} \cdot \frac{d \bar{e}}{dt}.$$

Since  $d \ell'(\mu^h | \bar{e}) / d \bar{e}$  should be negative (more care reduces insurance payments), any small tax that increases household efforts at accident avoidance will improve welfare. Moreover, the net social value of the tax change is just equal to the reduction in the ex-

24. Note that, for each  $\gamma$ , individuals choose to maximize their expected utility; but they do not take into account the effect of  $e^h$  on  $\bar{e}$  (which is negligible) and  $\gamma$ .

25. Note that as  $t$  changes, the optimal policy  $\mu^h$ , will change, but by the envelope theorem, this effect drops out. Also in this formulation, the change in the maximum expected utility of each household is the partial of the Lagrangian of the constrained household maximization problem.

pected level of casualty insurance payments. Again this is an observable consequence against which the efficiency of a tax intervention can be measured. (It is obvious that in a one-good economy, commodity taxes cannot be used to effect a Pareto improvement; those who have studied insurance markets in isolation of other markets—taking other prices as fixed—have, not surprisingly, come to the misleading conclusions that competitive insurance markets ought not to be interfered with as long as a competitive equilibrium exists; see, for example, Shavell [1979].)<sup>26</sup> The principle that emerges from (21) seems intuitive: commodities like fire extinguishers that decrease the frequency and size of insured against losses should be subsidized, while those, like alcohol, which increase the frequency and size of losses should be taxed. Arnott and Stiglitz [1986] have provided a general characterization of the set of optimal corrective taxes.

Note finally that because all individuals are identical, it is much easier to effect a Pareto improvement in this case than in the signaling and adverse selection models discussed earlier, where the government may face an informational problem concerning who should be compensated for any price change.

### E. Incomplete Markets

An economy without a full set of Arrow-Debreu contingent commodity markets is one in which many commodities (securities) are composites. When changes in demand change market prices, the nature of the composite product will often change. As a result,

26. If we assume that insurance can be made to depend on the *complete* vector of household consumption, equilibrium in a competitive insurance industry will imply that

$$\gamma(\mu^h, x^h, \bar{e}) = \epsilon(\mu^h | \bar{e}(x^h), x^h),$$

where the  $\bar{e}$  in question is now that of households with consumption vector  $x^h$ , since these households constitute a separate insurance class. Under these conditions

$$\frac{dR}{dt} = \sum_H \frac{d \epsilon(\mu | \bar{e}(x^h))}{d\bar{e}} \cdot \frac{d\bar{e}(x^h)}{dt},$$

where  $x^h$  is being held constant as taxes change. However, if taxes do not affect  $x^h$ , then they will not affect  $e^h$  and thus will have no impact on  $\bar{e}$ .

Therefore, where insurance premiums are conditioned on *all* components  $x^h$  which affect  $\bar{e}$ , tax interventions will not be able to improve overall consumer welfare. (The original competitive equilibrium may still not be Pareto efficient, but commodity taxes will not help. See Arnott and Stiglitz [1984].) The ultimate policy question is whether insurance firms can monitor individual household consumption levels or whether it is easier for the government to control overall consumption levels via taxes. (A similar but slightly more complicated analysis can be applied to the adverse selection case presented earlier.)

“quality” variable externalities will exist just as in the adverse selection case, and although the notion of “quality” is no longer unambiguous, small tax interventions will almost invariably exist that can improve an original market allocation. The initial allocation is not, therefore, a Pareto optimum.

A simple model of the phenomenon involved is one with two periods. Assume that, in period 2, the state of nature may take on one of  $k$  values. Assume further that there is a single store of value, denoted good zero, whose relative price in period 2 depends on the state of nature that materializes at that time.<sup>27</sup> Let an  $(n + 1)k$ -dimensional vector  $s = (s_1, \dots, s_k)$  denote the vector of price vectors of  $n$  period-two nonnumeraire commodities with  $s_{0k} \equiv 1$  for all  $k$ , in each of the period 2 states of nature. The value of this vector will depend upon market conditions in period 2, which depend, among other things, on taxes and the amount of the good zero available in period 2. If good zero is the only store of value, then a household’s expected utility at the beginning of period 2 depends on its holdings  $W_0^h$ , of this good at that time and the vector of prices  $s$ . For each  $W_0^h$  and  $s$ , there is a function  $V^h(W_0^h; s)$  which describes the maximum expected utility of household  $h$  in period 2. For concreteness,  $V^h$  can be written as

$$V^h(W_0^h; s) = \sum_k u_{2k}^h(x_k^{h*}; W_0^h, s_k) b_k,$$

where  $b_k$  is the probability that state  $k$  materializes. The vector  $x_k^{h*}$  is the consumption that maximizes the utility of household  $h$  during period 2 in state  $k$ . It is selected to maximize  $u_{2k}^h(x)$  subject to the constraint that

$$s_k \tilde{x}_k^h \leq 0,$$

where  $\tilde{x}_k^h$  is the individual’s (second period) net trade vector; for commodity zero,

$$\tilde{x}_{0k}^h = x_{0k}^{h*} - W_0^h,$$

while for the remaining commodities,

$$\tilde{x}_{j,k}^h = x_{j,k}^{h*} - W_{j,k}^h,$$

27. A more conventional approach would be to follow Diamond [1967] and Stiglitz [1972, 1982b], who assume that the investment good yields a random return. If there are grounds for government intervention in the more restrictive model used here (in which the “real” return to the investment goods is fixed at zero), then there are certainly grounds for government intervention in the more general model.

where  $W_{jk}^h$  is the individual's second-period endowment vector in state  $k$ .

Looking forward from the beginning of period 1, we shall assume that a household's two-period expected utility is the sum of its expected utilities in period 1 and period 2 separately. Formally,

$$(22) \quad u^h(W_0^h; s) = u_1^h(\bar{W}^h - W_0^h) + V^h(W_0^h; s),$$

where  $\bar{W}^h - W_0^h$  denotes consumption in period 1 of the store of value good, where  $\bar{W}^h$  is his total initial endowment of that good. (We ignore first-period consumption other than of good zero.) Households choose  $W_0^h$  to maximize two-period utility.

Now consider the impact of a small change in period 2 prices. It will lead to changes in  $W_0^h$  purchased and, by this means, to changes in the vector  $s$ .<sup>28</sup> In equation (22), the vector  $s$  enters the overall utility function directly as a kind of externality. Like the quality variable in the adverse selection example, it describes the "composition" of a ticket in a lottery. In this instance, the lottery is a subsequent value lottery instead of a quality lottery (and the individual is concerned with more than the mean value). Thus, changes in the "prices"  $s$  have real welfare effects.

Application of equation (13) to this simple model implies that a small change in taxes  $dt$  will have a net impact per unit tax,

$$\frac{dR}{dt} = \sum_H \sum_k \frac{dE^h}{ds_k} \frac{ds_k}{dt} b_k = \sum_k \left[ \sum_H \tilde{x}_k^h \cdot \frac{\lambda_k^h}{U_1^h} \right] \frac{ds_k}{dt} b_k,$$

where  $\lambda_k^h$  is the marginal utility of income to household  $h$  in state  $k$ .<sup>29</sup> Therefore, in general, there will exist taxes that can improve overall welfare.<sup>30</sup> Models that conclude otherwise typically impose conditions under which  $ds_k/dt = 0$  for all  $k$  or in which the pattern of prices that occurs across states of nature has no welfare consequences (e.g.,  $\sum_H \tilde{x}_k^h (\lambda_k^h / U_1^h) = 0$  for all  $k$ ). For example, Dia-

28. In addition, if there were a vector of consumption first period, it would lead to a readjustment of that vector, the effects of which net out.

29. An increase in  $s_k$  reduces utility in the  $k$ th state by  $x_k \lambda_k^h$ . To compensate requires a first-period increase in income of  $x_k \lambda_k^h / U_1^h$ .

30. It is worth noting that Pareto improvements can sometimes be effected by levying taxes or subsidies on variables that are not state contingent. (This may be important if, for instance, it is claimed that the reason that there is not a complete set of Arrow-Debreu securities is the unobservability by third parties, including the government, of the state.) Such is the case where the level of storage can be affected by taxes first period (which would arise if we had a vector of commodities the first period).

mond [1967] achieves this by having only a single good so that  $s_k = 1$  for all  $k$  under all circumstances. The conditions involved are very special ones.<sup>31</sup>

In general, tax changes induce changes in the distribution of prices across the states of nature, and this affects the ability of the limited number of markets that are available to provide their important risk-transfer–risk-sharing functions. Each individual trader, however, takes the price distribution as given, and hence, in making his decisions, ignores these considerations. Our results thus provide a negative answer to what has become a long line of research, to find general conditions in which, though there is not a complete set of markets, the competitive economy is still constrained Pareto efficient, constrained, that is, by the limitations on the available risk market.<sup>32</sup>

### F. Queue Rationing

When information is imperfect and search (transactions) is costly, the benefits and costs of entering a market often depend on variables other than price. For instance, the return to a worker entering the labor market depends on both the length of time that he has to search for a job as well as the wage he receives once he is employed. And the length of time that an individual has to search depends on the search activities of other individuals.

31. Note that if all individuals are identical,  $\bar{x}_k^i = 0$ , and the economy is constrained Pareto efficient (but then the risk markets serve no useful purpose, and no trade occurs on them). Note too that if individuals are risk neutral,

$$\lambda_k = U_i^h, \text{ so } \sum_H \bar{x}_k^i \frac{\lambda_k^i}{U_i^h} = \sum_H \bar{x}_k^i = 0 \text{ (by market clearing).}$$

Again, the absence of risk markets causes no problems, since risk markets are really unnecessary. More general conditions under which risk markets are redundant (and the market equilibrium is constrained Pareto efficient) are derived in Stiglitz [1982b] and Newbery and Stiglitz [1981, 1982].

32. Earlier studies [Stiglitz, 1972; Drèze, 1974; Hart, 1975] showed that with an incomplete set of markets, there could be multiple equilibria, some of which Pareto dominated others. The results reported here show that in general every equilibrium is Pareto inefficient—that (to use the distinction introduced in Stiglitz [1972]) there are marginal inefficiencies as well as (possibly) structural inefficiencies. Other studies identifying marginal inefficiencies include Stiglitz [1972, 1975b, 1982b], Loong and Zeckhauser [1981], and Newbery and Stiglitz [1981, 1982, 1983, 1984].

Still other studies, in particular that of Grossman [1977], have attempted to find a definition of constrained Pareto optimality such that the economy with limited risk markets is indeed constrained Pareto efficient. His Social Nash Optimality concept entails fixed transfers across individuals in the second period in different states. There appears to be no natural market interpretation of this constraint: the changes in prices induced by tax changes do entail changes in the relative magnitudes of the transfers.



Similarly, in product markets, queues (and other nonprice mechanisms) may often be an integral part of the process of balancing supply and demand. The length of a queue and associated waiting costs may again depend on the actions of other firms and individuals.<sup>33</sup> In both cases, there is an externality. The question is whether these externalities result in markets being Pareto inefficient. We now show how these externalities can be analyzed using the framework of this paper. The example we investigate involves queue rationing. The reasons for looking at queues are threefold. First, they have not been investigated as thoroughly as search equilibria.<sup>34</sup> Second, the structure of the models is quite general. And third, queue rationing equilibria usefully illustrate the set of circumstances in which competitive equilibria are Pareto efficient (in ways that most conventional search models do not) even when nonprice mechanisms are an important part of the market-clearing process.

Again, to facilitate the exposition, we shall use a very simple model. Let there be a single good, subscript 1. The "good" is supplied in  $N$  separate markets indexed  $i = 1, \dots, N$ , in each of which firms provide a different average waiting time. Consumers have rational expectations and know the probability distribution of waiting times for each type firm.<sup>35</sup> For simplicity, we assume that they are concerned only about the mean waiting time. An equilibrium set of prices equates supply and demand in each of these markets (as always, we ignore existence problems). Let

$$q_i \equiv \text{consumer price of the "good" in market } i = 1, \dots, N,$$

$$q = (q_1, q_N)$$

33. Similar externalities arise when firms must bear some part of the hiring and training costs of individuals, and individuals' quit rates depend on the actions of other firms. Still other search externalities that may be analyzed using our framework are those where the characteristics (quality) of individuals arriving at a firm are affected by the policies of other firms.

34. An exception is Truman Bewley's unpublished paper, "Equilibrium Theory with Transactions Costs."

35. Although this specification of "markets" may seem slightly unnatural, it is used to eliminate two obvious kinds of queuing inefficiency. First, having a separate price clear each waiting-time-defined market, we eliminate situations where time-on-queue substitutes for higher prices. Second, we eliminate situations, similar to the adverse selection or moral hazard cases analyzed above where consumers know the *average* waiting time (or processing rate) for a group of firms, but not the characteristics of individual firms. In our model it may be helpful to think of a firm's commitment to have an actual average waiting time equal to that of its waiting-time-defined market being enforced by a reputational mechanism.



$$\begin{aligned}
 p_i &\equiv \text{producer price in market } i = 1, \dots, N, \\
 T_i &\equiv \text{average waiting time for consumers in market} \\
 & i = 1, N, \dots, T = (T_1, \dots, T_N).
 \end{aligned}$$

Each of the  $i$  markets are assumed to be competitive with both firms and consumers taking prices as given, and consumers taking waiting times as given.

Households will be assumed to divide up their purchase flows among the several markets. Let

$$\begin{aligned}
 x^h &\equiv (x_1^h, \dots, x_N^h) \equiv \text{vector of purchases by household } h, \\
 x &= \sum_H x^h.
 \end{aligned}$$

Household utility will be assumed to depend on  $x^h$  and, also, implicitly on the waiting time associated with  $x^h$ .<sup>36</sup>

Each firm produces output using a single machine characterized by an output rate per unit time  $y^f$ . Machines break down in any given market period with a probability  $(1 - r^f)$ , where  $r^f$  is a machine's "reliability," and if they do so, we assume that they produce nothing for the period in question. The cost of a machine is a firm-specific function  $c^f(r^f, y^f)$  of its output rate and reliability.

At the beginning of each period, consumers go to a particular market and select a firm. If the firm's machine is functioning, they look at the length of the queue and decide whether or not to wait (knowing the firm's processing rate). If the firm's machine has broken down, consumers select a new firm, and for simplicity, we assume that they do so costlessly until they join a queue.

Since firms in market  $i$  are committed to provide a waiting time  $T_i$ , their average process rates will have to be adjusted to meet this requirement, given the average rate of customer arrivals. That rate depends, in turn, on the reliabilities and processing rates of the machines of all firms in the  $i$ th market, since firms that have nonfunctioning or slow machines will tend to pass their customers on to others. The relationship even for this simple

36. For instance,

$$\begin{aligned}
 u^h &\equiv u^h(x^h, w - p \cdot x^h, L - T \cdot x^h) \equiv \text{utility function of household } h, \\
 \text{where} \\
 w &\equiv \text{total supply of labor} \equiv \text{labor income,} \\
 L &\equiv \text{nonworker hours.}
 \end{aligned}$$

Waiting times  $T$  will be assumed to be rates per units consumed which implies that there is a standard order quantity.

model will be a complex one, but, in general, it will take the form,

$$(23) \quad y^f = \Psi^f(\bar{y}_i, \bar{r}_i; T_i),$$

where  $\bar{y}_i$  is an  $F_i$ -dimensional vector of the processing rates of the  $F_i$  firms in the  $i$ th market and  $\bar{r}_i$  is an  $F_i$ -dimensional vector of reliabilities.

The average profit of an individual firm can, then, be written as

$$\Pi^f = p_i y^f r^f - c^f(r^f, y^f),$$

which is maximized subject to the constraint of equation (23). Substitution from (23) into the profit expression yields a reduced-form profit function that now depends on the externality or  $z$ -variables  $\bar{y}_i$  and  $\bar{r}_i$ . In general, the service rate required to attain a given mean service time depends on the actions of other firms in the market.

Notice that in this formulation, since individuals care only about mean service times, and these are "priced" by the market, externalities enter only through the profit function. Consequently, application of equation (13) yields

$$(24) \quad \frac{dR}{dt} = \sum_i \sum_{F_i} (p_i r^f - c_y^f) \left( \frac{\partial \Psi^f}{\partial \bar{y}_i} \cdot \frac{d\bar{y}_i}{dt} + \frac{\partial \Psi^f}{\partial \bar{r}_i} \cdot \frac{d\bar{r}_i}{dt} \right),$$

where  $c_y^f$  is the marginal cost of additional processing capacity to firm  $f$  (note that  $p_i r^f \neq c_y^f$  at the maximum profit, since firms must still meet a service time requirement).<sup>37</sup> Not only can, in general, a tax-subsidy scheme effect a Pareto improvement, but (24) shows that the appropriate direction of government policy can readily be determined by examining the impact of taxes on the service patterns facing firms and, in particular, on whether average extra processing capacity produces expected revenue below or in excess of its marginal cost.

We can also identify the special cases in which the market is Pareto efficient. Assume that firms could fix the arrival rates of consumers on their queues. This eliminates the spillover ex-

37. As usual, these are total derivatives. We consider here only small changes; note that for large changes, some firms may decide to change the market in which they enter; that is, firms must choose among all possible markets, the one that maximizes their profits. Though by the envelope theorem, the direct effects of these changes can be ignored, the discontinuities in the number of machines serving any particular market (which may result) imply that the relevant functions may not be continuous. We ignore these problems.

ternalities from the actions of other firms. Then, assuming that consumers may search costlessly among firms to find a queue to which they will be admitted (we are not examining search externalities here), there are no externalities and the market is Pareto efficient.<sup>38</sup>

### III. FURTHER EXTENSIONS AND APPLICATIONS

Beyond the examples discussed in the previous section, the general approach that we have developed in this paper has a variety of other applications and easy extensions, providing insights in a variety of phenomena. In this section we briefly outline some of the more important of these.

#### A. *Self-Selection Constraints*

Since the Rothschild-Stiglitz [1976], Wilson [1977], and Salop and Salop [1976] analyses of competitive equilibrium with self-selection constraints, the self-selection model has been used to investigate a variety of markets (insurance markets, labor markets, capital markets). Most of these studies were limited to a single market, taking all prices as given. The self-selection equilibrium, when it existed, was characterized as the allocation that maximized the welfare of the low-risk (high-ability, high-productivity) individual subject to the self-selection constraints being satisfied.<sup>39</sup> The equilibrium thus appeared to be (constrained) Pareto efficient. But as long as the self-selection constraints themselves can be affected by relative prices,<sup>40</sup> there exist taxes that can effect a Pareto improvement. Thus, in the education model if bright individuals use fewer pencils in going to school than do less bright individuals (they can do the necessary calculations in their head), then a tax on pencils has a differential effect on low- and high-ability individuals. Since the self-selection constraints represent a big wedge in the economy, it is not surprising that introducing a small wedge (in the pencil market) which reduces

38. Notice how restrictive these assumptions are. As both the search literature and these examples demonstrate, when nonprice processes play an important role in balancing supply and demand, the Pareto optimality of the "market" outcome will be unlikely.

39. Also, implicitly, the highest risk (lowest ability) individuals obtain what they would have obtained were they the only individuals in the market.

40. What is required is that the change in prices affect different types of individuals differently: there would almost always seem to be some commodity for which this is true.

the magnitude of the big wedge (the self-selection constraint) may be desirable. (Formally, the effects of self-selection constraints can most easily be analyzed within our model by embedding them into a "derived" utility function, in a manner analogous to how we analyzed moral hazard.)

A special application of the self-selection model that has received considerable attention recently is that where workers are uninformed concerning the state of nature; self-selection constraints are used to induce firms to tell truthfully the state of nature. (See Grossman-Hart [1983], Azariadis and Stiglitz [1983], and Stiglitz [1984] and the papers cited there.) The implicit contract equilibrium, with asymmetric information, is in general, not Pareto efficient.

### *B. Moral Hazard and Incentives*

The general set of issues discussed in subsection II.D arises not only in formal insurance markets, but also in a variety of other contexts in which there is implicit insurance, in which individuals do not bear the full costs of their actions. One well-studied example arises in economies with sharecropping [Stiglitz, 1974]. Braverman and Stiglitz [1982] have argued that in this context, the externalities across markets<sup>41</sup> may be so large and important that they are effectively internalized, through the interlinking of land, labor, and product markets. Similar effects arise in labor contracts in general, and managerial contracts in particular, when workers are not paid on a strictly piece rate basis. Moral hazard issues also arise in capital markets where both effort and risk-taking decisions may be affected [Stiglitz and Weiss, 1981].

### *C. Unemployment Equilibria*

Whenever the terms of a contractual arrangement affect the productivity of a worker (the riskiness of loan, etc.) either through selection or incentive effects, then there may exist equilibria that are not market clearing.<sup>42</sup> The informational problems imply that, in general, Pareto efficiency may entail unemployment; nonetheless, the market equilibrium is not, in general, a constrained

41. These arise from the effect of prices of credit or other commodities on effort exerted by the tenant; alternatively, the terms of the tenancy contract may affect default probabilities.

42. For a survey of these theories see Stiglitz [1985].

Pareto optimum. It should be noted, however, that the approach developed here does not apply directly to this problem, since we have assumed here that markets clear (see Greenwald and Stiglitz [1985]).

#### *D. Rationing Equilibria*

Again, it is easy to use our analysis to show that, in general, if there are distortions in the economy, caused by either commodity taxes, imperfect information, or incomplete markets, rationing may be desirable. Consider the effects, at a given set of prices, of a ration so large that only one individual (or a few) is affected adversely by it. The direct loss in welfare is negligible: at the margin, the individual is just indifferent to buying the last unit anyway. But the indirect effects, via prices, on the other distortions may be such as to make the rationing desirable. It might be argued that the resulting price changes are small and, thus, their consequences are negligible. However, as Appendix I demonstrates, this is not the case.

#### *E. Other Government Policies*

Rationing is but one example of a government policy, other than uniform taxes, which may be used to effect Pareto-improving price and other changes in the presence of distortions. Price effects must be taken into account in designing other government policies as well. Thus, the optimal supply of public goods may no longer be described by the Samuelson condition of the sum of the marginal rates of substitution equaling the marginal rate of transformation: the effect of a marginal increase in the supply of the public good on all relative prices must be assessed.

Similarly, if the government has imposed an optimal income tax, whether differential commodity taxes will effect a Pareto improvement can be analyzed directly within our framework by embedding the self-selection constraints into the utility functions, and analyzing the effect of price changes on the associated implicit externalities. (Our analysis thus can be used to provide an interpretation of the Atkinson-Stiglitz [1976] results, which give conditions under which no differential commodity taxation is desirable: see Stiglitz [1982a].)

#### *F. Prices Conveying Information*

There have been several recent studies focusing on the role that prices play in conveying information, say about the state of

nature. For instance, in the Grossman-Stiglitz model [1976, 1980], as more individuals become informed, the price distribution changes and becomes more informative. Our analysis can again be used to show that the competitive equilibrium is not (constrained) Pareto efficient: not only do various tax policies affect the ability of the economy to share risks (as described earlier) but also affect the information available to each individual, and this too acts like a  $z$ -variable, except in the unusual case where essentially any set of equilibrium prices is fully informative.

### *G. Large Welfare Consequences of Small Inefficiencies*

Our analysis in Appendix I demonstrates how a small perturbation to the economy can have significant general equilibrium welfare effects, when there are already distortions in the economy. The perturbations we focused on in the body of the paper were government induced. But there is nothing in the mathematics that requires this. Thus, consider the consequences of one firm not adjusting some control variable in response to a disturbance to the economy. The welfare loss to the firm of this seemingly slight irrationality is negligible; however, with existing distortions, the welfare loss to the economy will, in general, not be. (See Akerlof and Yellen [1984].)

### *H. Other Multipliers*

This example illustrates that there may be “multiplier” effects in the presence of distortions. The total welfare loss may be a large multiple of the welfare loss to any individual. The analysis of this paper has focused on welfare effects, partly because the envelope theorem enables considerable simplification. Our model can, of course, be directly applied to illustrate other possible multipliers; any perturbation will not only have a direct effect, but also the standard indirect effect through prices (which, for stable systems, usually reduce the magnitude of the direct effect), and an externality ( $z$ -) effect; in a variety of situations the latter may reinforce, rather than dampen, the direct effect.

## IV. CONCLUDING REMARKS

We conclude with some general remarks concerning our approach to the study of externalities. In several of our examples we were able to relate the appropriate direction of government policy to some simple, in principle observable, parameters. On

the other hand, we have considered relatively simple models, in which there is usually a single distortion (one kind of information imperfection, one kind of market failure). Though the basic qualitative proposition, that markets are not constrained Pareto efficient, would obviously remain in a more general formulation, the simplicity of the policy prescriptions would disappear. Does this make our analysis of little policy relevance? The same objection can, of course, be raised against standard optimal tax theory. (Some critics might say, so much the worse for both.) Though simple expositions of optimal tax theory often focus on the case of independent demand curves, in the general case, one needs to know all the cross elasticities of demand, and these are seldom available. What is worse, if one abandons the unrealistic assumption of the standard optimal commodity tax formulation (e.g., Diamond-Mirrlees [1971], with their assumption of 100 percent pure profits taxes, no restrictions on commodity taxation, and no (progressive) income tax), then the informational requirements on the government are even greater.

We believe, however, that in the case of the inefficiencies we have discussed here, there are some circumstances in which certain effects may be dominant, allowing the derivation of meaningful policy prescriptions, and there may be other circumstances in which all that is required is a reduced-form (general equilibrium) derivative, which it may be easier to obtain than to derive the underlying structural parameters. Thus, though there may be a complicated set of indirect effects from the imposition of a tax on alcohol, one might suspect that these indirect effects are outweighed by the direct effects associated with lower accident rates; and to assess whether taxation of alcohol is desirable, all that one needs to know is the net effect on accident rates.

It should be emphasized that none of the effects we have discussed depend on there being a finite number of individuals. It is sometimes thought that in "large" economies, pecuniary effects can be ignored, since the action of any individual has a very small effect on price. Although it is true that in large economies, the action of an individual has a very small effect on price, the change in the price affects a large number of individuals. The total welfare effect is the product of the magnitude of the change in the price, times the number of individuals who are affected. We show in Appendix I that this product does *not* go to zero as the size of the economy gets larger. Pecuniary effects do not matter in the standard competitive model simply because there are no



distortions; if there are distortions—imperfect information, incomplete markets, etc.—they matter, regardless of the size of the economy.

Last, we had considerable difficulty in choosing a title for this paper. One suggested title was “Externalities in Imperfect Economies.” This had one advantage over the title chosen: as should be clear from the analysis, our results apply to more than just the problems raised by imperfect information and incomplete markets. We rejected it, however, for two reasons. First, referring to economies with incomplete markets and imperfect information as “imperfect” seems to be wrong: we do not refer to economies in which inputs are required to produce outputs as “imperfect”; and the costs of obtaining information and running markets are no less real costs than other forms of production costs.

Second, the title seems to trivialize our results. It hardly seems surprising that there exist government interventions which can effect a Pareto improvement in an economy with externalities, and other imperfections. Nor should it come as much of a surprise that imperfect information and incomplete markets cause “problems.” Our results do, however, run counter to much of (at least the older) folk-wisdom. This suggested that although an economy with, say, imperfect information would not do so well as one with perfect information, this was an irrelevant comparison. The relevant comparison had to take these costs of information into account; when this was done, it was suggested (though not proved) that the efficiency of the competitive economy would be re-established. We hope this paper will have laid to rest this heuristic argument.

The paper thus casts a new light on the First Fundamental Theorem of Welfare Economics asserting the Pareto efficiency of competitive equilibrium. The theorem is an achievement because it identifies what in retrospect has turned out to be the singular set of circumstances under which the economy is Pareto efficient. There is not a complete set of markets; information is imperfect; the commodities sold in any market are not homogeneous in all relevant respects; it is costly to ascertain differences among the items; individuals do not get paid on a piece rate basis; and there is an element of insurance (implicit or explicit) in almost all contractual arrangements, in labor, capital, and product markets. In virtually all markets there are important instances of signaling and screening. Individuals must search for the commodities that they wish to purchase, firms must search for the workers who



they wish to hire, and workers must search for the firm for which they wish to work. We frequently arrive at a store only to find that it is out of inventory; or at other times we arrive, to find a queue waiting to be served. Each of these are "small" instances, but their cumulative effects may indeed be large.

We have constructed a general model which shows that in all of these circumstances, Pareto improvements can be effected through government policies, such as commodity taxes. Our methodology not only identifies the presence of inefficiencies, but also enables us to identify both the appropriate direction of policy intervention and observable measures of their successful application.

#### APPENDIX I

In order to investigate the nature of pecuniary externalities in the traditional sense, the natural starting point is to examine the impact of a small "balanced budget" shift in excess demand.<sup>43</sup> Let

$$d\bar{v}_0 \equiv (dv_1^0, dv^0),$$

where

$$\begin{aligned} dv_1^0 &\equiv -q \cdot dv^0 \equiv \text{shift in demand for the numeraire good,} \\ dv^0 &\equiv (N - 1) \text{ vector of shifts in demand for the } N - 1 \\ &\quad \text{nonnumeraire goods.} \end{aligned}$$

The shift  $d\bar{v}_0$  may be ascribed either to a shift in the demand of a single household or to entry of a new household. An analogous shift with  $dv_1^0 = -p \cdot dv^0$  could be defined and ascribed to a change in behavior by the universe of firms.

If taxes are unchanged, the resulting change in market prices is

$$dp = dq = J^{-1} \cdot dv^0,$$

where

$$J \equiv \left[ \frac{dx_j}{dp_k} - \frac{dy_j}{dq_k} \right], j, k = 2, \dots, N \equiv \text{Jacobian}^{44} \text{ of the} \\ \text{vector of nonnumeraire excess demands.}$$

43. Only balanced budget shifts make sense if we are considering changes in equilibrium allocations. An unbalanced shift in excess demand would preclude the existence of a new equilibrium.

44. Since  $dp_k = dq_k$  in the present instance, it makes sense to talk about this "Jacobian" without treating the  $p$  and  $q$  vectors separately.

We assume that the excess demand functions are differentiable and that  $J$  is nonsingular at the initial equilibrium.

The change in income necessary to maintain the utility level of household  $h$  in the face of a change in price  $dp = dq$  is

$$\frac{dI^h}{dp} = E_q^h + E_z^h \left( \frac{dz^h}{dp} + \frac{dz^h}{dq} \right) - \sum_F \alpha^{hf} \left( \pi_p^f + \pi_z^f \left( \frac{dz^f}{dp} + \frac{dz^f}{dq} \right) \right).$$

Summation over all households and recognition that  $\pi_p^f = y^f$ ,  $E_q^h = x^h$ ,  $\sum_H \alpha^{hf} = 1$ , and  $\sum_F y^f = \sum_H x^h$  yields a total net change in government income compensation,

$$\sum_H \frac{dI^h}{dp} = \sum_H E_z^h \left( \frac{dz^h}{dp} + \frac{dz^h}{dq} \right) - \sum_F \pi_z^f \left( \frac{dz^f}{dp} + \frac{dz^f}{dq} \right).$$

The total change in the government surplus (once these compensations are paid) is

$$\frac{dR}{dp} = t \cdot \frac{dx}{dp} - \sum_H E_z^h \left( \frac{dz^h}{dp} + \frac{dz^h}{dq} \right) - \sum_F \pi_z^f \left( \frac{dz^f}{dp} + \frac{dz^f}{dq} \right).$$

At an initial tax level of zero this becomes

$$\frac{dR}{dp} = - \sum_H E_z^h \left( \frac{dz^h}{dp} + \frac{dz^h}{dq} \right) - \sum_F \pi_z^f \left( \frac{dz^f}{dp} + \frac{dz^f}{dq} \right).$$

As a function of the initial change in excess demand, the net change in the government surplus<sup>45</sup> is

$$(A1) \quad \frac{dR}{dv^0} = \frac{dR}{dp} \cdot \frac{dp}{dv^0} = (\Pi^P - B^P) \cdot J^{-1},$$

where

$$\begin{aligned} \Pi^P &\equiv \sum_F \pi_z^f \left( \frac{dz^f}{dp} + \frac{dz^f}{dq} \right), \\ B^P &\equiv \sum_H E_z^h \left( \frac{dz^h}{dp} + \frac{dz^h}{dq} \right), \end{aligned}$$

45. The expression in equation (A-1) below ignores the externalities generated by changes in consumption that result from compensating government income transfers. This is not done because the changes in question are negligible; they are not negligible. Rather it is done to avoid keeping track of transfer-related externalities that add greatly to the notational burden without affecting the basic substance of the analysis. For rigor we could assume that (1) consumption and production of the numeraire good generates no externalities and (2) households are constrained to consume their compensating allotments of the numeraire good. Also we assume that the original shift affects no  $z$ -variables. Alternatively, the derivatives can be interpreted in the manner suggested in footnote 8.

This represents the net social impact of the initial change in price and, thus, the "pecuniary" externality<sup>46</sup> associated with original change in demand  $d\bar{v}^0$ .

It only remains to be shown that  $dR/dv^0$  does not vanish as the number of households becomes large. To do this, let

$$\begin{aligned}\eta_m &\equiv \text{fraction of households of type } m = 1, \dots, M \\ &\quad (\text{i.e., } \eta_m H = \text{number of households of type } m) \\ \eta_l &\equiv \text{number of firms of type } l = 1, \dots, L \text{ per household} \\ &\quad (\text{i.e., } \eta_l H = \text{number of firms of type } l)\end{aligned}$$

Since  $dz^f/dp$  and  $dz^f/dq$  ought not to be influenced by the number of households,<sup>47</sup>

$$\Pi^P \equiv \sum_L H \cdot \pi_z^l \left( \frac{dz^l}{dp} + \frac{dz^l}{dq} \right) = H \cdot \hat{\Pi}^P,$$

where

$$\begin{aligned}\hat{\Pi}^P &\equiv \sum_L \pi_z^l \left( \frac{dz^l}{dp} + \frac{dz^l}{dq} \right), \\ \pi_z^l &\equiv \pi_z^f \text{ for firms of type } l, \\ \frac{dz^l}{dp} &\equiv \frac{dz^f}{dp}, \frac{dz^l}{dq} = \frac{dz^f}{dq} \text{ for firms of type } l.\end{aligned}$$

The matrix  $\hat{\Pi}^P$  will not change with the number of households  $H$ . Similarly,

$$B^P = H \cdot \hat{B}^P,$$

where

$$\hat{B}^P \equiv \sum_M E_z^m \left( \frac{dz^m}{dp} + \frac{dz^m}{dq} \right),$$

and  $\hat{B}^P$  should be invariant to changes in  $H$ .

46. This effect differs from the tax effects of the body of the paper in that the  $dz/dp, dz/dq$  terms differ from the  $dz/dt$  terms. However, in both cases, externalities will not matter either when  $\pi_z^l$  and  $E_z^h$  are zero for all households and firms or when the  $z$ 's are not affected by changes in market prices (other cases are fortuitous). If  $dz^f/dt$  and  $dz^h/dt$  are nonzero, then as a rule  $(dz^f/dp + dz^f/dq)$  and  $(dz^h/dp + dz^h/dq)$  will be nonzero. Thus (again in general), the conditions under which taxes can lead to Pareto-improving allocations are precisely circumstances under which "pecuniary" externalities do not net out.

47. Clearly in some cases through crowding or other effects increases in the numbers of agents will themselves intensify the impact of a price change on particular externalities. Equally clearly we want to focus on cases where this does not happen. For instance, if  $z$  is the quality of air that is affected by the total level of consumption of some commodities, then replicating households but dividing the size of each household proportionately will, with homothetic preferences, lead to the appropriate limit for our purposes.

On the other hand, the Jacobian of the excess demands

$$\begin{aligned} J &= \left[ \frac{dx_i}{dq_j} - \frac{dy_i}{dp_j} \right] = \left[ \sum_H \frac{dx_i^h}{dq_j} - \sum_F \frac{dy_i^f}{dp_j} \right] \\ &= H \left[ \sum_M \eta_m \frac{dx_i^m}{dq_j} - \sum_F \eta_l \frac{dy_i^l}{dp_j} \right] = H \cdot \hat{J}, \end{aligned}$$

where

$$\hat{J} \equiv \left[ \sum_M \eta_m \frac{dx_i^m}{dq_j} - \sum_F \eta_l \frac{dy_i^l}{dp_j} \right],$$

which should be invariant to changes in  $H$ . The inverse of the Jacobian,  $J^{-1}$ , can now be written as

$$J^{-1} = 1/H \cdot \hat{J}^{-1},$$

which does go to zero as  $H$  increases. This reflects the fact that as the number of agents increases, the impact on prices of any single agent goes to zero.

However,

$$\begin{aligned} \frac{dR}{dv^0} &= (\Pi^P - B^P) \cdot J^{-1} = H(\hat{\Pi}^P - \hat{B}^P) \cdot \frac{1}{H} \cdot \hat{J}^{-1} \\ &= (\hat{\Pi}^P - \hat{B}^P) \cdot \hat{J}^{-1}, \end{aligned}$$

which is invariant to the number of households. Thus, "pecuniary" externalities vanish in atomistic economies only when  $\hat{\Pi}^P$  and  $\hat{B}^P$  are zero, which occurs, in turn, only in the absence of non-pecuniary externalities.

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