Price Discrimination and Social Welfare

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The effect on social welfare of third-degree price discrimination was first investigated by Joan Robinson (1933). Richard Schmalensee (1981) has recently reexamined this question and presented several new results. In particular, he noted that a *necessary* condition for price discrimination to increase social welfare—defined as consumers' plus producers' surplus— is that output increase.

Schmalensee established this result only in the case of independent demands and constant marginal costs. However, it turns out to be true in much more general circumstances. In this paper I show how simple methods from duality theory can be used to establish this result and several other new results on the welfare effect of price discrimination.

I. A Reservation Price Model

Before proceeding to an examination of price discrimination in a general context, it is worth pausing to consider the special case of a reservation price model. I will describe the model in the context of discrimination by age—as in senior citizen discounts or youth discounts—but several other interpretations are possible. Assume that we have a set of consumers of different ages, and that one unit will be demanded by the consumers of age a if the price facing these consumers, $p(a)$, is less than or equal to $r(a)$, the reservation price of these consumers. Suppose that the slope of $r(a)$ is of one sign, which without loss of generality we take to be negative. For simplicity, it is assumed that costs are zero, or equivalently, that constant marginal costs are incorporated into the definition of $r(a)$.

Suppose first that the monopolist must choose one price p_0 that will apply to all consumers. Then the maximization problem facing the monopolist is to chose a_0 to solve:

$$\max r(a_0)a_0.$$

Now suppose that the monopolist is allowed to price discriminate; that is, he can choose critical ages a_1, a_2 and prices p_1, p_2 such that the consumers younger than a_1 face price p_1 and consumers between a_1 and a_2 face price p_2. The problem facing the monopolist now is to solve:

$$\max r(a_1)a_1 + r(a_2)(a_2 - a_1).$$

In this model it is easy to see that consumers' plus producers' surplus is given by the area below the reservation price function, as depicted in Figure 1. Thus the total welfare rises when price discrimination is allowed if and only if total output goes up. And, as shown below, output must always rise in this sort of model.

FACT 1: *If $r(a)$ is a decreasing function, then output and thus welfare must increase when price discrimination is allowed.*

PROOF:

Assume not so that $a_0 > a_2$ and thus: $-r(a_0)a_1 > -r(a_2)a_1$. By profit maximization: $r(a_0)a_0 \geq r(a_2)a_2$. Adding these two inequalities together, and adding $r(a_1)a_1$ to each side of the resulting inequality gives

$$r(a_1)a_1 + r(a_0)(a_0 - a_1) > r(a_1)a_1 + r(a_2)(a_2 - a_1),$$

which contradicts profit maximization.

This result easily generalizes to the choice of many regimes of price discrimination as well: allowing more price discrimination al-

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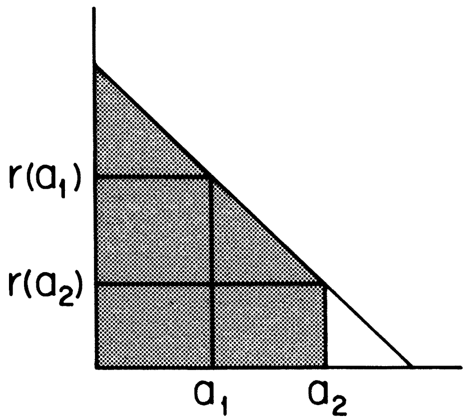

FIGURE 1. SURPLUS IN RESERVATION PRICE MODEL

ways increases output and welfare. As the number of prices increases to infinity, we converge to perfect price discrimination and thus maximal social welfare.

In this model we have a very simple story about price discrimination: price discrimination always increases output and an increase in output is always associated with an increase in welfare. But the reservation price model is a very special sort of demand structure and it is worth investigating whether these results carry over to more general demand specifications. As Schmalensee shows, in general, output and welfare may increase or decrease when price discrimination is allowed, although an increase in output remains a necessary condition for welfare increase. This result provides an observable criterion for when welfare has gone down under price discrimination, but how can we recognize those circumstances in which welfare has increased? I provide some answers to this question and related questions below.

II. Quasi-Linear Utility and Consumers' Surplus

I want to continue to use the classical measure of consumers' plus producers' surplus, and the most general preference structure for which that is possible is that of quasi-linear utility, which is also known as the case of "constant marginal utility of income." For this class of preferences it is well known that not only does consumer's surplus serve as a legitimate measure of individual welfare, but also that the individual consumers' utility functions can be added up to form a social utility function, so that aggregate *consumers'* surplus is also meaningful. For a discussion of consumers' surplus and indirect utility, see my 1984 book (ch. 7). These observations imply that we can treat the aggregate demand function as though it were generated by a representative consumer with an indirect utility function of the form:

$$V(\mathbf{p}, y) = v(\mathbf{p}) + y.$$

The aggregate consumer's income, y, is composed of some exogenous income which we take to be zero and the profits of the firm. Thus the appropriate form of the social objective function becomes:

$$V(\mathbf{p}, y) = v(\mathbf{p}) + \pi(\mathbf{p}).$$

By Roy's law the demand for good i is given by the negative of the derivative of $v(\mathbf{p})$ with respect to p_i—since the marginal utility of income is one. Thus the integral of demand is just $v(\mathbf{p})$. It follows that the above expression is nothing but the classical welfare measure of consumers' plus producers' surplus.

As a general principle, it is easier to differentiate to find demands than to integrate to find surplus; thus starting with the properties of the indirect utility function rather than the demand functions tends to simplify most problems in applied welfare economics. The most important property for our purposes concerns the curvature of the indirect utility function. The indirect utility function is always a quasiconvex function of prices, but in the case of quasi-linear utility, it is not hard to show that it is in fact a *convex* function of prices. (Proof: the expenditure function is $e(\mathbf{p}, u) = u - v(\mathbf{p})$ and it is necessarily a concave function of prices.)

III. Upper and Lower Bounds on Welfare Change

I turn now to the welfare effects of price discrimination for demand structures generated by quasi-linear utility. I start by describing a general result about such demands which can then be specialized in a number of ways. Consider an initial set of prices $\mathbf{p}^0$ and a final set of prices $\mathbf{p}^1$, and let $c(\mathbf{x}(\mathbf{p}^0))$ and $c(\mathbf{x}(\mathbf{p}^1))$ denote the total costs of production at the two different output levels associated with the price vectors $\mathbf{p}^0$ and $\mathbf{p}^1$. Let $\Delta\mathbf{x}$ denote the vector of changes in demand (i.e., $\Delta\mathbf{x} = \mathbf{x}(\mathbf{p}^1) - \mathbf{x}(\mathbf{p}^0)$), and let Δc denote the change in the total costs of production.

FACT 2: *The change in welfare*, ΔW, *satisfies the following bounds*:

$$\mathbf{p}^0\Delta\mathbf{x} - \Delta c \geq \Delta W \geq \mathbf{p}^1\Delta\mathbf{x} - \Delta c.$$

PROOF:

Since the indirect utility function is a convex function of prices, we have:

$$v(\mathbf{p}^0) \geq v(\mathbf{p}^1) + \mathbf{D}v(\mathbf{p}^1)(\mathbf{p}^0 - \mathbf{p}^1)$$

where $\mathbf{D}v(\mathbf{p})$ stands for the gradient of $v(\mathbf{p})$. Using Roy's law, and rearranging:

$$\mathbf{x}(\mathbf{p}^1)(\mathbf{p}^0 - \mathbf{p}^1) \geq v(\mathbf{p}^1) - v(\mathbf{p}^0) = \Delta v.$$

The change in profits is given by

$$\mathbf{x}(\mathbf{p}^1)\mathbf{p}^1 - \mathbf{x}(\mathbf{p}^0)\mathbf{p}^0 - \Delta c = \Delta\pi.$$

Adding these expressions together we have

$$\left[\mathbf{x}(\mathbf{p}^1) - \mathbf{x}(\mathbf{p}^0)\right]\mathbf{p}^0 - \Delta c$$
$$= \mathbf{p}^0\Delta\mathbf{x} - \Delta c \geq \Delta v + \Delta\pi = \Delta W.$$

The other bound can be derived in a similar manner.

Now think of the n goods as being one good sold in n different markets and produced at constant marginal cost. I want to compare a uniform pricing policy to a policy of price discrimination. Making the necessary substitutions in the bounds given in Fact 2, we have the following:

FACT 3: *Let* $\mathbf{p}^0 = (p_0, \ldots, p_0)$, $\mathbf{p}^1 = (p_1, \ldots, p_n)$, *and let c be the constant level of marginal costs. Then the bounds on welfare change become*

$$(p_0 - c)\sum_{i=1}^{n} \Delta x_i \geq \Delta W \geq \sum_{i=1}^{n} (p_i - c)\,\Delta x_i.$$

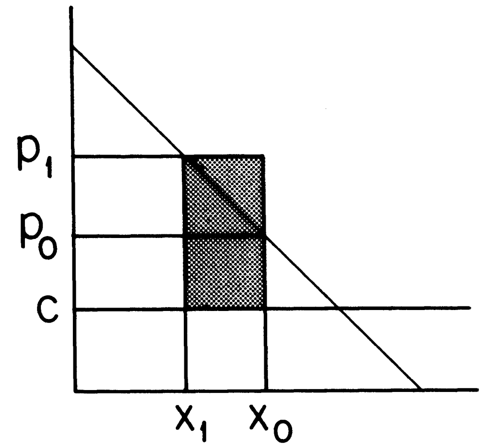

FIGURE 2. BOUNDS ON WELFARE CHANGE IN SINGLE MARKET

Note that the upper bound in Fact 3 immediately gives Schmalensee's result that an increase in output is a necessary condition for welfare to increase. The lower bound in Fact 3 was not discussed by Schmalensee. It implies that if the profitability of the new output exceeds the profitability of the old output, *valued at the new prices*, then welfare must have risen at the discriminatory equilibrium. This is basically a revealed preference relationship.

Both of these facts hold in complete generality, for independent and dependent demands, as long as one is willing to assume quasilinear utility; that is, that aggregate consumers' surplus serves as an acceptable welfare measure. The bounds have a simple geometric interpretation in the case of a single demand curve which is given in Figure 2. However, it is worth emphasizing that these results are purely statements about demand and utility functions and hold for arbitrary configurations of prices. The fact that the prices are chosen by a profit-maximizing monopolist has not been used in their derivation.

IV. Bounds on Welfare Change with Optimal Price Discrimination

I now ask what results can be derived that *use* the conditions implied by *profit-maximizing* price discrimination. Let us specialize the notation above to consider only three prices, the initial price p_0 that is charged in both markets, and the final prices p_1 and p_2 that are profit-maximizing prices in their respective markets. We also continue to suppose that the good is produced at constant marginal cost c.

Fact 3 holds for all prices and all demand structures. If we consider only profit-maximizing prices and restrict ourselves to the textbook case of independent demands, we can apply the standard marginal revenue equals marginal cost formulas to find:

FACT 4: *If demand functions are independent, welfare is bounded by*

$$\frac{c[\Delta x_1 + \Delta x_2]}{\varepsilon_0 - 1} \geq \Delta W \geq \frac{c\Delta x_1}{\varepsilon_1 - 1} + \frac{c\Delta x_2}{\varepsilon_2 - 1},$$

where $\varepsilon_0, \varepsilon_1, \varepsilon_2$ are the (absolute values of the) respective elasticities of demand, evaluated at p_0, p_1, and p_2.

This result may be of use if one has estimates of the elasticities of demand in the various submarkets. However the independent demand case is rather restrictive. Profit maximization *alone* yields the following sufficient condition for a welfare increase.

FACT 5: *A sufficient condition for welfare to increase under profit-maximizing price discrimination is that*

$$(p_0 - c)[x_1(p_0, p_0) + x_2(p_0, p_0)]$$

$$> (p_1 - c)x_1(p_0, p_0) + (p_2 - c)x_2(p_0, p_0).$$

PROOF:

By profit maximization at (p_1, p_2) we have

$$(p_1 - c)x_1(p_1, p_2) + (p_2 - c)x_2(p_1, p_2)$$

$$\geq (p_0 - c)x_1(p_0, p_0) + (p_0 - c)x_2(p_0, p_0).$$

Combining this with the hypothesis and rearranging, we have $(p_1 - c)\Delta x_1 + (p_2 - c)\Delta x_2 > 0$. By Fact 3 this yields a welfare increase.

The interesting thing about Fact 5 is that it only involves a condition on the nondiscriminatory levels of output. If you can forecast the prices that would be charged under discrimination and those prices satisfy the condition given in Fact 5, you can be assured that welfare will rise when discrimination is allowed.

It might be worthwhile to give an example of how these bounds can be used to verify that a welfare increase or decrease has occurred. The simplest example is the case of linear demands described by Schmalensee. If both markets are served in the single price regime, then it is easy to show by direct calculation that total output with discrimination is the same as in the single price regime. Hence, as noted by Schmalensee, welfare must decline when discrimination is allowed.

However, suppose we are in a situation where market 2 is not served in the single price regime. Then when discrimination is allowed, $p_1 = p_0$, $\Delta x_1 = 0$, and $\Delta x_2 > 0$. By Fact 3 welfare must increase. Note also that in this situation the sufficient condition given in Fact 5 is satisfied as an equality.[1]

Thus Fact 3 verifies that welfare will increase when price discrimination is allowed in the linear demand case if a new market is served. However, Fact 3 also shows that for *arbitrary* independent demands, welfare goes up if a new market is served when price discrimination is allowed. The argument is simply that of the above paragraph: $\Delta x_1 = 0$ and $\Delta x_2 > 0$, so welfare must increase.

These examples give some intuition for the case where both markets are served in both the discriminatory and nondiscriminatory regimes as in Figure 4. What is needed for welfare to increase when price discrimination

[1] Of course, total output rises as well. The reader might wonder what is wrong with the "direct calculation" mentioned above. The problem is that what economists call "linear" demand curves are not really linear functions; instead they have the form: $Q = \max\{A - BP, 0\}$.

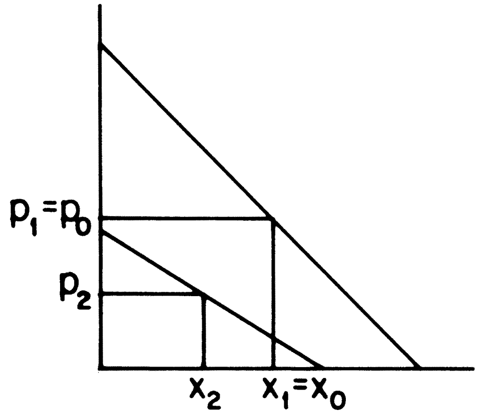

FIGURE 3. INCREASE IN WELFARE (BOUNDARY CASE)

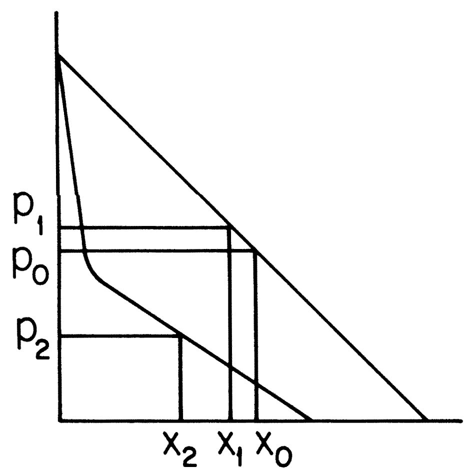

FIGURE 4. INCREASE IN WELFARE (INTERIOR CASE)

is allowed is that one of the markets has small demand over the price range where the other market has large demand.

Another test case for the bounds is the reservation price model described in Section I. Here we should think of each consumer as being a different market with demand function $x_a(p)$. If there are a_0 consumers purchasing the good in the single-price regime and $a_2 > a_0$ under price discrimination, then we know that $\Delta x_a = 0$ for $a \leq a_0$ and $\Delta x_a = 1$ for $a_2 \geq a > a_0$, which by Fact 3 implies welfare must increase when discrimination is allowed.

The bounds can also be used to show that marginal cost pricing and perfect price discrimination are welfare optima in the reservation price model. For if price equals marginal cost, the upper bound on welfare change is zero. And if each consumer is being charged his reservation price, then Δx_a is either 0 or -1 which implies the upper bound in nonpositive.

The welfare bounds given above take a nice form if we are willing to make curvature assumptions on the demand functions. Let us restrict ourselves to the case of independent demands and focus on the market for good 1. Then the argument of Fact 2 implies that the welfare effect of a price change of good 1 is bounded by $(p_0 - c)\Delta x_1 \geq \Delta W_1 \geq (p_1 - c)\Delta x_1$. Suppose that the demand for good 1 is a concave function of its own price. Then we have $\Delta x_1 \geq x_1'(p_1)(p_1 - p_0)$. Combining these two inequalities we have $\Delta W_1 \geq (p_1 - c)x_1'(p_1)[p_1 - p_0]$. The first-order conditions for profit maximization imply that $(p_1 - c)x_1'(p_1) + x_1(p_1) = 0$. Substituting we have $\Delta W_1 \geq x_1(p_1)(p_0 - p_1)$. If both markets have concave demand curves we can write:

$$\begin{aligned}\Delta W &\geq x_1(p_1)(p_0 - p_1) + x_2(p_2)(p_0 - p_2) \\ &= p_0[x_1(p_1) + x_2(p_2)] \\ &\quad - [p_1 x_1(p_1) + p_2 x_2(p_2)].\end{aligned}$$

Add and subtract $(p_0 - c)[x_1(p_0) + x_2(p_0)] - c[x_1(p_1) + x_2(p_2)]$ to get $\Delta W \geq (p_0 - c)\Delta x - \Delta\pi$, where Δx is the total change in output and $\Delta\pi$ is the total change in profits. Thus the change in welfare is at least as large as the change in profit valued at the old prices minus the change in actual profit. Or, to put it another way, $\Delta x > \Delta\pi/(p_0 - c)$ is a sufficient condition for welfare to increase when price discrimination is allowed if all demand curves are independent and concave. Combining this with Fact 3 we can conclude:

FACT 6: *If all demand curves are independent and concave the welfare bounds can be written as*

$$(p_0 - c)\Delta x \geq \Delta W \geq (p_0 - c)\Delta x - \Delta\pi.$$

Note that Facts 5 and 6 use profit maximization at p_1 and p_2, but do *not* use profit maximization at p_0. Thus these results are independent of firm behavior at the nondiscriminatory equilibrium.

If the demand curves are concave and convex (i.e., linear), then the inequality in Fact 6 becomes an equality so that $\Delta W = -\Delta\pi$. Thus in the case of linear demands, the change in welfare is exactly the negative of the change in profits. Of course this can also be verified by direct calculation.

V. More General Cost Structures

The above results were all derived in the case of constant marginal cost but they can be partially extended to the case of increasing marginal costs; that is, the case of a *convex* cost function. By the standard convexity inequality:

$$\mathbf{Dc}(\mathbf{x}(\mathbf{p}^1))\,\Delta\mathbf{x} \geq \Delta c \geq \mathbf{Dc}(\mathbf{x}(\mathbf{p}^0))\,\Delta\mathbf{x}.$$

Combining this with the inequality given in Fact 2 we have

$$[\mathbf{p}^0 - \mathbf{Dc}(\mathbf{x}(\mathbf{p}^0))]\,\Delta\mathbf{x} \geq [\mathbf{p}^1 - \mathbf{Dc}(\mathbf{x}(\mathbf{p}^1))]\,\Delta\mathbf{x}.$$

Again, these are general bounds which hold for all pairs of price vectors $\mathbf{p}^0$ and $\mathbf{p}^1$ as well as for arbitrary convex cost functions; in particular the cost function can be a function of the vector of outputs rather than just the total output. Thus the bounds can be useful in more general contexts. For example, they give a simple proof of the optimality of marginal cost pricing in the presence of convex costs: if $\mathbf{p}^0 = \mathbf{Dc}(\mathbf{x}(\mathbf{p}^0))$ then any movement from $\mathbf{p}^0$ must decrease social welfare.

If costs depend only on total output, denoted by x_0 and x_1, and $\mathbf{p}^0$ is a vector of constant prices p_0 as above, we can write these bounds as

$$[p_0 - c'(x_0)] \sum_{i=1}^{n} \Delta x_i \geq \Delta W \geq \sum_{i=1}^{n} [p_i - c'(x_1)]\,\Delta x_i.$$

Thus in the case of increasing marginal costs, Schmalensee's proposition still holds: price must be greater than marginal cost at the nondiscriminatory price, so an increase in output is still a necessary condition for welfare to increase.

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