THE RATIONALIZATION OF SUCCESSION TAXATION*

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I. THE PROBLEM

Present methods of levying succession taxes in the form of estate, inheritance, and gift taxes leave much to be desired. Relatively minor changes in the form of transmission of property often produce substantial differences in tax. In order to take advantage of these differences, individuals are often led to dispose of their property in ways other than those that would obtain in the absence of such avoidance opportunities. The patterns of ownership thus encouraged by the operation of the tax are not necessarily better, and in fact are often considerably worse, considered from the standpoint of the community at large, than the patterns that would be selected in the absence of such arbitrary pressures.

The root of the difficulty is that the tax is ordinarily computed on each transfer separately with very little if any reference to the relation of that transfer to past and future transfers of the same or equivalent property. The taxpayer is thus under considerable pressure to provide for the transfer of his property to the ultimate beneficiaries with as few taxable intervening transfers as possible. Property is accordingly bequeathed directly to children, grandchildren, and great-grandchildren rather than in the more normal sequence of transfer first to the widow, then to the children, and in turn to the grandchildren and great-grandchildren. The testator frequently may attempt to restrict the control of the remote heirs over the property thus directly bequeathed to them by the setting up of various forms of trust; in addition, by setting up trusts for the benefit of minors and even of unborn individuals, the number of taxable transfers may be still further reduced. In some states this may go to the extent of removing the corpus of the estate from further succession taxation for extended periods. The forms of property thus promoted have serious effects on the economic life of the community, in that they multiply the overhead of institutional investment, reduce the amount of capital available for speculative ventures, and are frequently less suited to carrying out the desires of the testator than less involved forms of devolution that might have been selected in the absence of tax pressures.

* A less technical discussion of the proposal presented here with emphasis on practical problems will be found in William Vickrey, "An Integrated Succession Tax," *Taxes*, Vol. 22, August, 1944, pp. 368–374. In addition to the method proposed here, the suggestion is there made that as a less precise alternative there might be imposed a cumulative inheritance tax with rates graduated according to the difference in age between donor and recipient.

II. A METHOD OF SOLUTION

If these effects are to be avoided, the tax burden must be made to depend only on the ultimate distribution of the property and not on irrelevant or trifling differences in the method by which this ultimate distribution is achieved. At least such differences as arise should if possible be related to acceptable social goals and not be merely the haphazard result of taking the line of least resistance in the manner of assessing the tax. This condition may be met by making the total succession-tax collections from all taxpayers, cumulated with interest, depend only on the way in which inherited wealth is distributed at a given time. To produce a tax that will call for further collections as wealth is transferred from one generation to the next, the younger the possessors of a given amount of wealth, the greater must be the total tax paid, and we may insert in the formula the age, or more conveniently the date of birth, of each owner of wealth as a factor which will determine this total tax. In the most general terms we can then put:

$$T_1 = T_1(t, b_1, b_2, b_3, \cdots, w_1, w_2, w_3, \cdots), \tag{1}$$

where T_1 is the present value of total tax receipts, t is the time, the b's are the dates of birth of the various property owners, and the w's are the amounts of wealth owned.

Then, when a transfer takes place, the amount of tax to be paid is obtained by taking the value of T_1 before the transfer and subtracting it from the value of T_1 corresponding to conditions as they exist after the transfer. Naturally, if the tax is to be workable at all, the tax to be paid on the occasion of a particular transfer must be calculable from the parameters relating to the donor and recipient only. This practical need suggests a formula of the form:

$$T_1 = \sum_n f(t, b_n, w_n). \tag{2}$$

If we compute the change in this function resulting from a given transfer, all the terms except those involving the donor and recipient will cancel out, leaving:

$$\Delta T_1 = f(t, b_d, w_{d1}) + f(t, b_r, w_{r1}) - f(t, b_d, w_{d0}) - f(t, b_r, w_{r0}), \tag{3}$$

where the subscript d refers to the donor, r to the recipient, 0 to the condition before the transfer, and 1 to that afterwards. We must also have the condition that there be no unaccounted-for funds, so that

$$w_{d0} + w_{r0} = \Delta T_1 + w_{d1} + w_{r1}. \tag{4}$$

From these two equations, given the initial conditions and either the gross cost of the transfer to the donor or the net addition to the inherited wealth of the recipient, the amount of tax and the remaining quantities can be computed.

The solution of these equations is implicit rather than explicit, however, and is not the type of straightforward computation that would be feasible in a tax law. To facilitate the computation of the tax we can eliminate ΔT_1 from the preceding equations and get:

$$\begin{aligned}(5)\qquad & w_{d0} + f(t, b_d, w_{d0}) + w_{r0} + f(t, b_r, w_{r0}) \\ & \qquad = w_{d1} + f(t, b_d, w_{d1}) + w_{r1} + f(t, b_r, w_{r1}).\end{aligned}$$

We can put

$$(6)\qquad g(t, b, w) = w + f(t, b, w),$$

and the above equation reduces to

$$(7)\qquad g(t, b_d, w_{d0}) + g(t, b_r, w_{r0}) = g(t, b_d, w_{d1}) + g(t, b_r, w_{r1}).$$

g is then the inherited wealth that a person would have had if there had been no succession taxes previously, and can appropriately be termed the "potential wealth" of a given individual at a given time. The above equation (7) shows that the total potential wealth of all individuals is unaffected by transfers, and a given transfer can be considered as the transfer undiminished of a given amount of potential wealth from one person to another.

The function $g(t, b, w)$ can now be tabulated, and the inverse function $w(t, b, g)$ obtained either by inverse entry of that table or from a separate tabulation. The tax can now be computed by first obtaining from these tables the potential wealth of the donor and recipient, and the potential wealth of the donor after the gross amount of the proposed transfer has been deducted from his w. From (7) we can then obtain g_{r1} and then by reference again to the tables, w_{r1} is determined. The tax to be paid is then obtained from (4). If the transfer is specified in terms of the net amount to be realized by the recipient, the process is the same except that w_{r1} is first obtained by adding the net gift to w_{r0} whence we get from the tables g_{r1}, from (7) g_{d1}, from the tables again w_{d1}, and from (4) ΔT_1.

III. NECESSARY CONDITIONS

It remains to determine the form of the function g so as to produce a behavior of the tax burden that conforms as nearly as may be to various desirable standards.

One of the conditions that must be satisfied by the tax is that there be no payment of tax when no transfers take place. Obviously if w is defined in such a way as to depend on saving or dissaving out of earned income (earned is here used in the sense of being derived directly or indirectly from personal activities as distinguished from normal interest on inherited wealth), a tax will be indicated every time there are such savings and a refund every time there are dissavings. If the tax is not collected on these occasions, it will then become impossible to satisfy the other essential condition that the tax burden be independent of the time at which a given transfer is made. One way of defining w which will give reasonably satisfactory results and still be independent of any saving or dissaving out of earned income is as the present value of all net gifts and bequests received less the present value of all gifts and bequests made and succession taxes paid. This implies of course that where bequests are made out of personally saved income, w may take on negative values. Treatment of such cases will be discussed later.

Complete invariance of the tax burden with respect to changes in time of transfer is almost impossible to achieve as long as there are income taxes that include as part of the base for graduated rates the interest on inherited wealth held and on wealth accumulated from personally earned income that is to be transferred to heirs. The tax on the interest earned by a given estate between two alternative dates of transfer will differ according to whether the estate is in the hands of the donor or the recipient, and the difference will depend not only on the size of the estate but on the income received by the two parties from other sources. Allowance for all of these factors would require a complicated computation for each tax assessment involving almost the entire history of the income and income tax of the two parties. Even if this computation could be made feasible, the net result would necessarily be merely a tax credit against succession taxes for income taxes on the interest on the wealth in question. The same result would be achieved more simply and directly by a substitution of a spendings tax for the income tax. In what follows it will therefore be assumed that the income tax does not enter the picture.

For purposes of simplification, we may make the substitutions

$$W = \frac{w}{A(t)}, \qquad T = \frac{T_1}{A(t)},$$

$A(t)$ being the amount of \$1 accumulated at compound interest from a fixed time $t=0$ to the time t (it is not necessary to assume at this point that the rate of interest be constant). W and T are then the inherited wealth and total tax revenue discounted to time $t=0$. They will both remain constant, as will the date of birth of the various taxpayers, as long as no transfers are made. Equation (4) retains its form in terms of the new variables:

$$W_{d0} + W_{r0} = W_{d1} + W_{r1} + dT, \tag{8}$$

and we may consider the function $g(t, b, W)$ without loss of generality in place of $g(t, b, w)$.

Let us consider first the transfer of a small sum dW (gross) to an individual of the same age but having a total wealth larger by DW than the donor. We then have:

$$\begin{aligned} &g(t, b, W) + g(t, b, W + DW) \\ &\qquad = g(t, b, W - dW) + g(t, b, W + DW + dW - d^2T), \end{aligned} \tag{9}$$

where d^2T is the tax to be paid, and is a second-order infinitesimal since it becomes small by reason of both the small transfer and the small differential in the wealth of the parties. At the limit, as dW approaches zero, we have[1]

$$g_W'(t, b, W)dW = g_W'(t, b, W + DW)(DW - d^2T), \tag{10}$$

and as DW approaches zero,

$$g_W'd^2T = g_{WW}''dWDW; \qquad \frac{d^2W}{dWDW} = \frac{g_{WW}''}{g_W'}. \tag{11}$$

To satisfy the condition that the tax be unaffected by the time of transfer, this last expression must be a function of W and b only. Accordingly, we may put

$$\frac{g_{WW}''}{g_W'} = h(b, W), \tag{12}$$

where h is an arbitrary function; integrating with respect to W, we get

$$\log (g_W') = H(b, W) + C(t, b), \tag{13}$$

where the two functions H and C are still arbitrary.

Consider again the transfer of a small sum dW to an individual of the same wealth but younger by db. We now have, similarly,

$$\begin{aligned} g(t, b, W) + g(t, b - db, W) &= g(t, b, W - dW) \\ &\quad + g(t, b - db, W + dW - d^2T), \end{aligned} \tag{14}$$

$$g_W'(t, b, W)dW = g_W'(t, b - db, W)(dW - d^2T), \text{ and} \tag{15}$$

$$\frac{d^2T}{dbdW} = -\frac{g_{bW}''}{g_W'}. \tag{16}$$

If in (13) we put $H = \log u$, $C = \log v$, (13) may be written:

[1] The notation g_W' is used for $\partial g/\partial W$; similarly $g_{bW}'' = \partial^2 g/\partial db W$ and $g_{WW}'' = \partial^2 g/\partial W^2$, etc.

$$g_W' = u(b, W)v(t, b). \tag{17}$$

From this we may write (16) in terms of u and v thus:

$$\frac{d^2T}{dbdW} = \frac{v_b'}{v} + \frac{u_b'}{u}\cdot \tag{18}$$

This must be a function of W and b only. Since u is not a function of t, v_b'/v must likewise not involve t. We may therefore put

$$\frac{v_b'}{v} = f(b), \quad \log v = F(b) + c(t), \text{ or } v = y(b)C(t). \tag{19}$$

Putting this back in (17), we have

$$g_W' = C(t)u(b, W)y(b); \tag{20}$$

and by integrating,

$$g = C(t)z(b, W) + k(b, t), \tag{21}$$

where C, z, and k are arbitrary functions of the variables indicated. This is the most general form for g that will satisfy the condition that the postponement of a transfer shall not affect the tax burden. We may for convenience select $k(b, t)$ such that $z(b, 0)=0$, without changing the complete function. Actually the terms $k(b, t)$ and $C(t)$ do not increase the variations possible in the behavior of the tax, for they cancel out in any computation of an actual tax. We may thus for convenience put $k=0$ and $C=1$ so that $g=z(b, W)$, and proceed to determine the function z so as to produce desired characteristics.

IV. OTHER DESIRABLE CONDITIONS

In determining $z(b, W)$ it is convenient to consider four limiting cases among the many types of transfers that might be made. Consider first the transfer of the entire estate of the decedent to a single heir having no other inherited wealth and younger than the decedent by db. We then have

$$z(b, W) = z(b + db, W - dT). \tag{22}$$

As db approaches zero,

$$0 = z_b'db - z_W'dT; \qquad \frac{dT}{db} = \frac{z_b'}{z_W'}\cdot \tag{23}$$

Second, consider the division of an estate equally among $(1+dn)$ heirs each younger than the decedent by db, the number being such that because of the splitting up of the estate into smaller units there will be no tax. We then have:

$$(24) \qquad z(b, W) = (1 + dn)z\left(b + db, \frac{W}{1 + dn}\right).$$

Again letting db approach zero, we have

$$(25) \qquad Wz_W'dn - z_b'db = zdn, \qquad \frac{dn}{db} = \frac{z_b'}{Wz_W' - z}.$$

Third, consider the transfer of a unit of wealth dW to an individual with a wealth smaller by DW and younger by db, such that there will be no tax. Then

$$(26) \qquad \begin{aligned} &z(b, W) + z(b + db, W - DW) \\ &\qquad = z(b, W - dW) + z(b + db, W - DW + dW); \end{aligned}$$

letting dW approach zero, we have

$$(27) \qquad z_W'(b, W) = z_W'(b + db, W - DW),$$

and letting DW likewise approach zero,

$$(28) \qquad 0 = z_{Wb}''db - z_{WW}''DW; \qquad \frac{DW}{db} = \frac{z_{Wb}''}{z_{WW}''}.$$

Finally we can consider the case already analyzed in (9) to (11), and (11) now becomes

$$(29) \qquad \frac{d^2T}{dWDW} = \frac{z_{WW}''}{z_W'}.$$

These relationships are, however, in terms of W which is the value of wealth discounted to a fixed time in the past. If the tax on a given type of transfer, for example of an entire estate of $1,000,000 to a single heir 30 years younger, is made a function of W alone, the exemptions and rate brackets will increase through time at compound interest, thus gradually lightening the tax, whereas the reverse is likely to be desired. There must of course be some such progression of these rates and brackets through the life of a given taxpayer if the invariance of the tax with respect to the time of transfer is to be maintained. However, this may be kept from being a secular lightening of the tax as a whole by relating the tax rates not to the value of wealth discounted to a fixed date but to the value of wealth discounted to the date of birth of the taxpayer, or to the date on which he attains a given age. To do this we put

$$(30) \qquad m = A(b)W, \qquad P(b, m) = z(b, W), \qquad x = A(b)T;$$

from these definitions we have

(31) $z_W' = A(b)P_m', \ z_{WW}'' = A(b)^2 P_{mm}'', \ z_b' = P_b' + imP_m',$ and

(32) $z_{Wb}'' = (P_{mb}'' + imP_{mm}'' + iP_m')A(b)$, where $i = i(b) = \dfrac{A'(b)}{A(b)}$ ·

We can redefine b as not necessarily the date of birth of the taxpayer, but any date bearing a fixed relationship to it; any uniformity of rates and exemptions will now be with reference to wealth discounted to time b. The precise definition of b is material only to the extent that the rate of interest i may be variable rather than a constant.

Restating the above four limiting transfers in terms of m, P, x, and i we now have

$$(33) \quad \frac{dx}{db} = A(b)\frac{dT}{db} = A(b)\frac{z_b'}{z_W'} = \frac{P_b' + imP_m'}{P_m'} = G(b, m).$$

G defines in a general way the scale of progression of the tax in the limiting case of transfers of entire estates to single heirs and is the function most nearly comparable to the present type of estate-tax schedule. It can be termed the *generating function*. Also,

$$(34) \quad \frac{dn}{db} = \frac{z_b'}{Wz_W' - z} = \frac{P_b' + imP_m'}{mP_m' - P} = s(b, m).$$

s defines the rate at which estates must be split up as they are passed on from one generation to the next in order to avoid tax, and may be termed the *secant*. Third,

$$(35) \quad \begin{aligned} \frac{dm}{db} &= A(b)\frac{dW}{db} = A(b)\frac{z_{Wb}''}{z_{WW}''} \\ &= \frac{P_{mb}'' + imP_{mm}'' + iP_m'}{P_{mm}''} = Q(b, m). \end{aligned}$$

Q defines the direction in which small gifts may be made without the payment of the tax and may be termed the *contour direction*. Finally,

$$(36) \quad \frac{d^2x}{dm^2} = \frac{1}{A(b)}\frac{d^2T}{dW^2} = \frac{1}{A(b)}\frac{z_{WW}''}{z_W'} = \frac{P_{mm}''}{P_m'} = M(b, m).$$

M can be termed the *slope* of the tax in the m direction; it relates to the tax on redistributions of wealth among individuals of the same age.

For a proper progression of the tax, G, s, Q, and M should all be positive throughout the range of operation of the tax, and in addition G_m' must be positive. Also

$$\frac{d}{dm}\left(\frac{G}{M}\right) = \frac{G_m'}{m} - \frac{G}{m^2}$$

should be positive if the average rate of tax is to increase with increasing wealth, and if possible G_{mm}'' should be positive also. It might be considered desirable to have G, s, Q, and M all functions of m only and independent of b, which would produce a tax that would be the same from one generation to the next.

It is necessary to provide some means of exempting transfers among persons of small wealth. Transfers from persons having wealth below such an exemption level and those above it cannot consistently be exempted, otherwise there would be an opportunity for avoidance through the use of these exempt persons as intermediaries. It is therefore necessary to provide some P for such persons to be used in case of transfers to or from persons above the exemption level. If the tax is to be consistent and continuous at the exemption level, the same P used for such taxable transfers should automatically result in no tax if used to compute a tax on transfers between persons below the exemption level. In the exemption region we must have therefore M and G both zero, or $P_{mm}''=0$, and $P_b'=-imP_m'$; the solution of these equations is $P=km/A(b)$. At the exemption level, which is the boundary between the exempt and taxable regions, we must have P continuous, otherwise there would be taxes of 100 per cent and over on some transfers. If therefore $m=E(b)$ is the exemption level, the function for the taxable region must also have this value along the boundary:

$$P(b,\ E) = \frac{kE(b)}{A(b)}\cdot \tag{37}$$

V. SOME DESIDERATA UNATTAINABLE

Consider first the possibility of putting $G=G(m)$. Then if we can assume that i is constant, the general solution of (33) will be

$$P = F(u), \quad \text{where} \quad u = b + \int \frac{dm}{G - im} \tag{38}$$

and F is an arbitrary function. Now if G is to be continuous, and positive everywhere above the exemption level, E must be the value for which $G(E)=0$, so that E must be a constant. $u(b,\ E)$ is then $b+C$, and we must have

$$F(b + C) = \frac{kE}{A(b)} = kEe^{-ib}.$$

To satisfy this condition we must put

$$F(u) = kEe^{-i(u-C)}. \tag{39}$$

But if we put (38) in (36), we get for M

(40)
$$M = \frac{\dfrac{F''}{F'} - G' + i}{G - im}$$

and from (39), $F''/F' = -i$, so that (40) reduces to

(41)
$$M = -\frac{G'}{G - im}\,.$$

Since G' and M must both be positive, G must be less than im. This condition rather drastically limits the scope of the tax, for it implies that the tax on an estate passed on to a single heir will be less than the interest accumulated on the estate over a period equal to the difference in their ages. For example, if an estate is passed on annually to an heir one year younger, the annual tax will be less than im.[2]

Even if the difficulties at the exemption level could be met in some other way, such as by eliminating the exemption entirely, or introducing a region above the exemption where G would be zero but M not zero, the solutions available under this heading are still too limited to be satisfactory. If the tax is to have a wider scope than that possible under the limitation that G must be less than im, then for some value of m, say r, $G(r) = im$. Now as m approaches r in either direction, if $G'(r)$ is finite, u approaches $-\infty$ independently of the constants of integration so that at $m = r$, F''/F' has a single value, say f, for all values of b. If M is to be positive in the neighborhood of r on both sides of r, the numerator in (40) must be zero at r, so that we must have $G'(r) = i + f$. Further, G' may not be greater than $G'(r)$ for values of m greater than r [where $(G - im)$ is positive], for were G' greater than $G'(r)$, for any

[2] We may in passing exhibit a sample function that fulfills all the desiderata except the essential one of being capable of imposing a tax as heavy as may be desired. Put $G = im + a/m - c$, and determine a and c so that $G'(E) = 0$ and $G(E) = 0$. Then

$$G = im + \frac{E^2 i}{m} - 2iE, \qquad G' = i - \frac{iE^2}{m^2}, \qquad \text{and} \qquad G'' = \frac{2iE^2}{m^3}\,.$$

G, G', and G'' are all positive for m greater than E.

$$u = b + \int \frac{dm}{iE\left(\dfrac{E}{m} - 2\right)} = b - \frac{m}{2iE} - \frac{1}{4i}\log\,(2m - E),$$

$$F(u) = e^{-iu} = e^{-ib}e^{m/2E}(2m - E)^{1/4} = P$$

$$s = i, \qquad M = \frac{m^2 - E^2}{Em(2m - E)}, \qquad M(E) = 0, \qquad Q = im.$$

Thus s, M, and Q are all positive for m greater than E, and are independent of b. M is continuous at the exemption level.

given value of m a b can be found such that $|F''/F'-f|$ will be less than any assignable quantity and in particular less than $G'(m)-G'(r)$, so that M would be negative. Similarly, G' must not be less than $G'(r)$ for any m smaller than r, where $(G-im)$ would be negative. These conditions again drastically limit the graduation of the tax. While it is possible to have a certain degree of graduation under these conditions, the average rate of tax will always be less than $G'(r)$, and the region for which $G=0$ must extend above $[r - ri/G'(r)]$.

We may also examine the possibility of making M independent of b. Integrating (36), we have

$$\log P_m' = \int M dm + \log F(b), \tag{42}$$

$$P = F(b) \int e^{\int M dm} dm + f(b) = F(b) I(m) + f(b), \tag{43}$$

where F and f are arbitrary functions and $I(m) = \int e^{\int M dm} dm$. Putting this result in (33), we obtain for G;

$$G = im + \frac{F'I + G'}{FI'}, \qquad G_m' = i + \frac{F'}{F} - M(G - im). \tag{44}$$

Here again, if G is ever to be greater than im, G' must be less than $i+F'/F$, while in regions where G is less than im, G' must be greater than $i+F'/F$. These limitations again stand in the way of a satisfactory graduation of the tax.

VI. DERIVING AN ACCEPTABLE FUNCTION

Finally try making s a function of m only. One form of the general solution of (34) is then

$$P = me^{-ib}e^{F(u)}, \tag{45}$$

where

$$u = b + \int \frac{dm}{(s - i)m}$$

and $F(u)$ is any arbitrary function. Putting this result in (33), we have

$$G = \frac{msF'}{s - i + F'} \cdot \tag{46}$$

If we are to have $G(E)=0$, we must have either $E=0$, $s(E)=0$, or $F'[u(b, E)]=0$. $E=0$ implies no exemption and is rejected. $s(E)=0$ implies either that E is a constant or that $s(m)=0$ over a range of values above the exemption. This latter condition in turn implies that over that same range either $P_m'=0$ or $G=0$, since from (33) and (34) we have

$$s = \frac{P_m' G}{m P_m' - P} \cdot \tag{47}$$

$P_m'=0$ involves indeterminateness of the tax, while $G=0$ reduces the tax in this range from a tax on succession to a tax on the redistribution of wealth. If on the other hand we take E as a constant, we must have

$$ke^{-ib} = P(E) = Ee^{-ib}e^{F(E)} \tag{48}$$

and thus $F(E)$ must be a constant. But

$$F\left(b + \int \frac{dm}{(s-i)m}\right)$$

can be a constant for varying values of b only if F is identically a constant or if $s=i$. Thus the only alternative left is to put $F'[u(b, E)]=0$. This is possible either if E is such a function of b that $u(b, E)$ is a constant, or if $s(E)=i$. It is desirable to keep the exemption constant if possible, and we shall therefore elect to put $s(E)=i$. Then if $s'(E)$ is finite (and also if s' approaches ∞ slowly enough as m approaches E) we have $u(b, E)=-\infty$ for all values of b.

It is then necessary but not sufficient to select F in such a way that $F'(-\infty)=0$. In addition, for $G=0$ we must have $F'/(s-i)=0$. The usual procedure of evaluating this fraction at the limit $m=E$ by differentiation of numerator and denominator fails here, for we get merely $F''/ms'(s-i)$ and so on *ad infinitum*. If, on the other hand we put $s=s'(E)\ (m-E)+i$ in the neighborhood of $m=E$, then in that neighborhood we have approximately

$$u = b + \int \frac{dm}{ms'(E)(m-E)} = b + \frac{\log\left(1 - \dfrac{E}{m}\right)}{Es'(E)} \cdot \tag{49}$$

If we put

$$a = Es'(E), \quad v = e^{au} = e^{ab}\frac{m-E}{m}, \quad \text{and} \quad H(v) = F(u),$$

then

$$F'(u) = H'(v)v'(u) = avH'(v) = aH'(v)e^{ab}\left(1 - \frac{E}{m}\right) \tag{50}$$

and

$$\frac{F'}{(s-i)} = \frac{aH'e^{ab}(m-E)}{s'(E)(m-E)m} = \frac{EH'e^{ab}}{m} \cdot \tag{51}$$

Then $H'(0)=0$ is the necessary and sufficient condition that $G(E)=0$.

In selecting a definite function for s, there are several limitations that must be observed. First, from (47) we have

$$sm = \frac{G}{1 - \dfrac{P}{mP_m'}}, \tag{52}$$

so that sm must be greater than G. Accordingly, if, as seems desirable, G/m is to increase without limit as m increases, s must likewise. Again from (46) we have

$$\frac{G}{m} = \frac{F'}{1 - \dfrac{i}{s} + \dfrac{F'}{s}}, \tag{53}$$

so that if F' had a finite upper bound, G/m would also; therefore we must have F' increasing without limit. On the other hand, since (53) may be rewritten

$$\frac{G}{m} = \frac{s}{1 + \dfrac{s-i}{F'}}, \tag{54}$$

if F' were to become infinite for any finite m, we should have $G=ms$, which from (52) would imply $P/mP_m'=0$, which is inappropriate.

Futher, let us put

$$u = b + S; \qquad S = \int_k^m \frac{dm}{(s-i)m}; \tag{55}$$

if $S(\infty)$ were finite and $F'[b_1+S(\infty)]=\infty$, then for each finite value of m there would be a b_2 such that $b_2+S(m)=b_1+S(\infty)$, and F' would be finite for a finite m. Thus we must have $S(\infty)=\infty$, while for the exemption to be a constant we must have $S(E)=-\infty$. Now since

$$\int_k^\infty \frac{dm}{m(\log m)^{1+j}} = \frac{(\log k)^{-j}}{j} \tag{56}$$

for all j greater than zero, then if $S(\infty)$ is to be infinite, $(s-i)/(\log m)^{1+j}$ must approach 0 as m increases, for any positive j. Again, since

$$\int_E^k \frac{dm}{m\left(\log \dfrac{m}{E}\right)^{1-j}} = \frac{\left(\log \dfrac{k}{E}\right)^j}{j} \tag{57}$$

for all j greater than zero, we must have, in case $S(E) = -\infty$, $(s-i)/\log (m/E)^{1-j}$ approach zero as m approaches E, for any positive j.

The simplest function that satisfies all these requirements for s is $s = i + a \log(m/E)$. For brevity let us put $L = \log (m/E)$. Then

$$(58) \qquad s = i + aL, \quad u = b + \frac{\log L}{a}, \quad v = e^{au} = Le^{ab}.$$

From (46) and (50) we have

$$(59) \quad G = \frac{amsvH'}{s - i + avH'} = \frac{am(i + aL)Le^{ab}H'}{aL + aLe^{ab}H'} = \frac{m(i + aL)e^{ab}H'}{1 + e^{ab}H'},$$

and it remains to determine H in such a way that $H'(0) = 0$, H' is never negative, and H is finite for finite v and has no upper bound.

The simplest function satisfying these conditions is $H = v^k$ where k is greater than 1. We have then finally $F(u) = H(v) = v^k$, and

$$(60) \qquad P = me^{-ib}e^{F(u)} = me^{-ib}e^{e^{kab}[\log (m/E)]^k}, \quad \text{or}$$

$$(61) \qquad \log \log \left(\frac{Pe^{ib}}{m}\right) = kab + k \log \log \frac{m}{E} \cdot$$

For brevity we can put $B = ke^{kab}$ and $P = me^{-ib}e^{(BL^k/k)}$. We then have

$$(62) \qquad G = \frac{BmL^{k-1}(aL + i)}{1 + BL^{k-1}},$$

$$(63) \qquad M = \frac{BL^{k-2}(k - 1 + L + BL^k)}{m(1 + BL^{k-1})},$$

$$(64) \qquad Q = im + aLm + \frac{aLm}{k - 1 + L + BL^k},$$

$$(65) \qquad G_m' = \frac{BL^{k-2}}{(1 + BL^{k-1})^2} [i(k - 1) + (i + ka)L + aL^2 + (i + a)BL^k + aBL^{k+1}],$$

$$(66) \qquad G_{mm}'' = \frac{BL^{k-3}}{(1 + BL^{k-1})^3} [i(k-1)(k-2) + (i + ka)(k-1)L + (ak)L^2 + i(k - 1)(-k)BL^{k-1} + (k - 1)(i + 2a - ka)BL^k + a(k + 1)BL^{k+1} + aB^2L^{2k}],$$

$$(67) \quad \frac{\partial}{\partial m}\left(\frac{G}{m}\right) = \frac{BL^{k-2}}{(1 + BL^{k-1})^2 m} [i(k - 1) + akL + aBL^k].$$

All of these quantities are positive throughout the applicable range for all values of k greater than 1, with the exception of G''. For G'' it will be noticed that if as L approaches zero, B increases in such a way that BL^k is kept constant, the term $i(k-1)(-k)BL^{k-1}$ eventually dominates the right-hand parenthesis in the expression for G'', and causes it to become negative. This is not serious, however, for it is shown that the average rate of tax, G/m is increasing throughout.

The simplest case occurs when $k=2$. We then have:

$$(68)\qquad \begin{aligned} P &= me^{-ib}e^{\frac{1}{2}BL^2}, \qquad G = \frac{mBL(aL+i)}{1+BL}, \\ M &= \frac{B(1+L+BL^2)}{m(1+BL)}, \end{aligned}$$

$$(69)\qquad G_m{}' = \frac{B[i+(2a+i)L+(a+aB+iB)L^2+aBL^3]}{(1+BL)^2},$$

$$(70)\qquad G_{mm}{}'' = \frac{B[i+2a-2iB+(2a+iB)L+3aBL^2+aB^2L^3]}{m(1+BL)^3},$$

$$\frac{\partial}{\partial m}\left(\frac{G}{m}\right) = \frac{B[akL+i(k-1)+aBL^2]}{m(1+BL)^2}.$$

It may be noted that for the year when $B=a/i$, G reduces to amL. G'' can have negative values only when B is greater than $\frac{1}{2}+a/i$ and L is less than 2. That G'' should be negative in such a limited region seems not too serious a defect, inasmuch as $(d/dm)(G/m)$ is always positive. Extensive search has failed either to discover alternative functions $H(v)$ and $s(m)$ which will produce a G'' everywhere positive, or to produce a proof that no such functions exist within the conditions laid down. Mathematically minded readers are invited to try their hand at the problem. For $k=2$, $G'(E)$ and $M(E)$ are positive, whereas for the sake of continuity and smoothness of graduation it would be somewhat preferable to have them both zero at this point, as actually does occur for k greater than 2. For k greater than 2 and less than 3, G'' is infinite at $m=E$, which is perhaps no serious objection. For k greater than 3, a high degree of continuity at the exemption level is obtained.

VII. TYPES OF TABLES

If the actual computation of the tax is to be from tables (and at least a table of logarithms, if no other, will be essential), the form of the function itself is perhaps of little importance unless it can be put in such a form that the computation can be reduced from reference to a double-entry table to reference to one or more single-entry tables.

We could, for example, stipulate a function reducible to the form $F(P)=B(b)+H(m)$. However, we should then have $G=im+B'/H'$; $G_b'=(G-im)B''/B'$; and we should have either G increasing with lapse of time for large estates while it decreased for small ones, or vice versa. It may be possible to find a function otherwise satisfactory that will lend itself to ready calculation from single-entry tables, but such a function is not obvious, at least, and the use of double-entry tables gives a flexibility that will have substantial other advantages. In any case, double-entry tables would probably be no great burden to the taxpayer, especially as it would be possible to take b at annual intervals and thus classify all taxpayers, ignoring fractions of a year; in this way interpolation would be necessary in only one direction and could be provided for by marginal factors similar to the present surtax rates. There would be a slight discrimination between transfers to or from those born just before and just after the beginning of the year, but this could have no tendency to cause wasteful reactions on the part of the taxpayer, as is the case with present discriminations, since the source of this discrimination is largely beyond the control of the taxpayer.

VIII. DISTRIBUTIONS FROM PERSONALLY SAVED WEALTH

We come now to determine P for cases where there is a distribution out of the net savings of the taxpayer, so that his bequests and gifts exceed his inherited wealth and W and m become negative. Here there is little question of the relation of the tax to other previous transfers, but purely a question of the proper level of tax to be levied on the creation of an estate, after which the tax on the transfer of that estate to subsequent heirs will follow the standard formula. One possibility, for example, would be to set $P(-m)=-e^{-ib}m$, continuing in effect the function used over the range of the exemption. However, this would mean that the tax on the transfer of individually saved wealth to a given person would be the same regardless of the age of the transferor, and would become progressively heavier rather rapidly in successive generations. Thus it will probably be desirable to provide some form of "tax credit" in the case of such transfers as an offset to the total tax to have been paid as indicated by the age and potential wealth of the recipient. On the other hand it is essential that this credit be independent of the amount of inherited wealth transmitted through the particular saver, lest the independence of the tax burden from the mode of transfer be destroyed. The tax credit must accordingly depend only on the amount of personally saved income and the age of the saver.

General conditions of symmetry here would point to the use of $P(b, -m)=-P(b, m)$ as probably a suitable function. The fact that the same tables would be used for positive and negative values of m would be an advantage. Further, consider a married couple of the same age, where the wife has an inheritance. She may either save it and transfer it to their children, or use it to pay the household expenses while the husband saves a corresponding amount of his earnings which would normally have been spent for this purpose, and makes the transfer instead of the wife. The use of this function for negative values of m will keep the tax the same in either case or in any intermediate case provided only that the ages of the husband and wife are the same. If their ages are not the same, it will probably be extremely difficult in any case to take account of such indirect transfer between members of one household; however, in the typical case in which the gainfully employed spouse is also the older, such indirect transfers would be from the younger to the older spouse and would not produce a saving of taxes. Thus there will seldom arise a tax incentive for disturbing the normal state of affairs.

IX. APPROXIMATE ALLOWANCE FOR INCOME TAXES

It has been assumed thus far that no income tax was in effect and that an estate if left untouched would accumulate at compound interest at a constant rate i. Secular variations of course do take place in the rate of interest, and to retain the invariance of the tax with respect to mode of transfer under these conditions it is necessary to make corresponding variations in the functions P, G, M, s, and Q. To allow for such universal secular variations in the interest rate we have merely to put, instead of (58) and (60):

$$(71) \qquad P = \frac{me^{e^{kab}[\log\,(m/E)]^k}}{A(b)}, \quad s = a \log\left(\frac{m}{E}\right) + i(b), \text{ and so on.}$$

With an individual income tax in effect, however, the rate at which an estate will accumulate will vary not only with such secular variation in the gross interest rate, but also with variations in the rates of income tax paid on this interest by various holders. Thus if an estate is invested in such a way as to yield 5 per cent gross, and is in the hands of a person whose income is such that the rate of income tax is 40 per cent, the net yield and rate of growth of the estate is 3 per cent, while if the same estate were in the hands of an individual with a larger income from other sources so that the income tax rate were 60 per cent, the net yield and rate of growth would be only 2 per cent. As the rate of growth of an estate will depend not only on the size of the estate but also on the other income of the holder, it will be extremely difficult if not practically impossible to make a complete allowance for this factor in a succession tax.

An approximate allowance can, however, be made, if we can assume some approximate relation between the inherited wealth of an individual and his total income. In accordance with some such assumption, we can compute what the net rate of increase of an estate in the hands of an individual with the assumed income will be, considering the income from the estate as being the income in the top brackets and subject to the highest rates. By applying this rate of increase year by year to legacies and inheritances, we can arrive at an amount that can be presumed to approximate the actual inherited wealth w at a given time. Corresponding to this amount we can arrive at an amount m that he would have had to have started with at date of birth (or other arbitrary age) to have accumulated the amount w at the present time without inheritances or bequests or gifts. This m will then be constant as long as there are no transfers and as long as the income bears the assumed relation to the inherited wealth. Since P depends only on b and m, P will likewise be constant under these conditions.

To determine m in practice from the available data, however, will require recourse to another set of tables for the determination of m from W and b and of W from w and t, where W is now defined as the amount at a given fixed base date that would accumulate to w at the time in question if subject to the income-tax rates on income associated with the various w's of the intervening years. On the whole it would probably simplify the work of the taxpayer if instead of these subsidiary calculations, P is determined directly from a triple-entry table as a function of t, b, and w. Actually only one value of t would be pertinent for the computation of the tax at any one time, and the taxpayer could be directed to use simply a double-entry table $P_t(b, w)$. It would probably be sufficient to set up such double-entry tables at annual intervals. While using only discrete annual values for t will reintroduce a slight incentive for making transfers at the beginning or end of the year, depending on the relative wealth or income of donor and recipient, this incentive should be sufficiently slight to cause no great trouble. However, should it be felt desirable to eliminate even this slight tendency and thus distribute transfers more normally over the year, quarterly or more frequent tables can be prepared, or taxpayers can be required to interpolate or make some other adjustment between one year and the next. At first glance it seems doubtful that any such refinement as this will be needed.

In practice it may not be necessary for the taxpayer to compute the initial w of the donor and recipient directly from the record of past bequests and inheritances, for it may prove simpler and more satisfactory merely to carry forward the value of P from a previous return, P being constant between transfers, and compute the initial w's from these initial P's by reference to tables. Because of the possibility of thus simplifying the process of obtaining the initial w's, triple-entry tables may be more advantageous than double-entry tables even if the adjustment for the income tax were not a factor.

X. MODIFYING THE INCOME TAX

There remains as long as we retain the present form of income tax a bias in favor of transfer through intermediaries that have no source of income other than the interest on the estates held. The prime type of such intermediary is the trust. As it is one of the primary purposes of the present analysis to discover a method of assessment that will eliminate bias in precisely this direction, such a plan can be considered only partially successful. Complete success will require rather drastic modification of the income tax itself.

One method of adjusting the income tax so as to eliminate this bias would be to eliminate the interest on inherited wealth from the regular income-tax base, and subject it to a separately graduated income tax for which an exact allowance can be made in the succession-tax tables. This could be done by subtracting an amount equal to iw from the income-tax base, and computing a separate tax on this iw. To avoid producing an incentive for an individual having savings out of personally earned income to dispose of his property as soon as possible and thus obtain for the interest on this sum the favorable status of interest on inherited wealth, it would be necessary to provide that where w is negative, an amount iw must be added to the regular income and the tax computed on the combined amount, from which tax a credit equal to the tax at the special rates on the amount iw would be deductible as a tax credit. If the rates on the amount iw are the same as the regular graduated rates, the only difference being the separate graduation, then this would have the net effect of pushing the regular income into higher surtax brackets. It is doubtful whether any such scheme of basing the income tax in part on income no longer in the hands of the taxpayer would be readily acceptable. Moreover instances would occur where through dissipation of the estate the amount iw exceeded the entire income of the taxpayer, in which case a tax computed on the excess would have to be considered as a tax credit against the tax computed on the amount iw. Again the necessary complications will prove confusing and possibly irritating to the taxpayer. Actually the net result of all this complication would be a final tax burden that would not be so very different from that imposed by a spendings tax. On the whole it would be much simpler and more effective to go all the way to a spendings tax, in which case no involved scheme of integration of the two taxes would be necessary.

XI. RATE CHANGES

It is generally considered proper that succession-tax rates should change somewhat less frequently than income-tax and other tax rates, because of the occasional nature of the tax. But the very nature of the tax here proposed makes it imperative that there be a degree of permanence in the rate structure far greater than any heretofore achieved. Yet it cannot be expected that any rate schedule enacted will remain the same in perpetuity, and some provision for the alteration of rates will be necessary. Of course, if alterations take place so far in advance that no alteration of the schedule takes place for values of b and m pertaining to transfers already consummated, then there is no problem. This is indeed the ideal way of changing the schedule. But it implies a degree of patience on the part of legislators searching for methods of adjusting immediate tax receipts that may be hard to obtain, even though succession taxes must of necessity play a very minor role in the total fiscal picture.

If alterations take place in schedules after transfers involving these parts of schedules have taken place, either the invariance of the tax with respect to modes and times of transfer must be impaired or a direct or indirect *ex post facto* tax or refund must be imposed or allowed on these transactions that have already been consummated. If this *ex post facto* tax is imposed indirectly on the occasion of further transfers, there will remain a differential in the tax in the case where the sum transferred is spent without further transfer. It does not seem practical to impose what would amount to a species of capital levy at the time of a change in the schedule, and it would seem that this differential at least must remain. One method of levying this supplementary tax on the occasion of further transfers would be to require the use of the old schedule for the relationship between the initial P and the initial w and the new schedule for the relationship between the final P and the final w. The chief objection to this is that it would sometimes result in a tax of over 100 per cent in the case of some small transfers, thus leaving nothing at all to the beneficiary. It would be difficult to grant a general relief in such cases without at the same time providing an opportunity for transfers to be made designed to qualify for the relief and thus reintroduce undesirable repercussions. Some gradual method of transition from the old function to the new is required. For such cases a special function might be defined as follows:

$$ {}_iP = P_0 + k\,{}_2P_w'(b, t, w_0)(w - w_0), \tag{72} $$

where P_0 is the initial P carried forward from a previous return, ${}_2P$ is the new schedule, and k an arbitrary constant. For w greater than w_0, a value of k greater than 1 would be used, while for w less than w_0, a different value of k less than 1 would be stipulated. In either case ${}_iP$ is to be used only so long as it is less than ${}_2P$. For the first transfer after a change in the rate schedule, w_0 can be computed from the P_0 carried forward from the previous return. However, where the transition to the new schedule takes more than one transfer, the w_0 for the second transfer will either have to be computed from the final w_1 of the previous transfer by means of a special table showing $w(W, t)$ and $W(w, t)$ (W being unchanged between transfers) or by computing ${}_2P(t_1, b, w_{12})$ from the tables, and finding w_{21} so that ${}_2P(t_2, b, w_{21})$ produces the same P, w_{12} being the final w of the previous transfer and w_{21}, the initial w of the transfer under consideration. For other purposes of the computation, the same intermediate P carried forward from the previous return would be used, however.

XII. INITIATION

The inauguration of the new method of taxation could be considered as merely a special case of a change in the rate schedule. However, on the one hand there has been no invariance of the tax burden in the past and therefore no particular reason to insist on extending the invariance achieved by the new method to past bequests, since they will not be influenced by the change in any event, while at the same time it is particularly undesirable to continue the old and less satisfactory method to any considerable extent any longer than is necessary. Yet at the time of inauguration of the new form of tax there will be owners of wealth with wide variations in age and in sources from which the wealth was acquired. Records showing sources of wealth will not always be available, even where property was acquired fairly recently, for federal estate- and gift-tax records do not show in detail or with any uniformity the beneficiaries of such transfers. It would probably be necessary to assume unless it could be shown otherwise that all disposition of wealth held at the time of inauguration of the tax was made out of personally saved wealth. In general it would be to the benefit of the taxpayer to have all his wealth considered as personally saved and to conceal any gifts or bequests received in the past. In the face of this tendency and in the absence of any uniform and universal records covering any extended period in the past, the most satisfactory procedure will be to assume all property acquired before the enactment of the law to be personal savings, and set the initial P for the first transfer of all taxpayers under the new law at zero.

XIII. SUMMARY

We thus have a reasonably practical method of computing a tax on gratuitous transfers such that, except for the effect of the income tax, the tax burden is substantially independent of the channels or methods by which transfers are made. Even the differences arising from the income tax may be eliminated by modifying that tax. The computations required of the individual taxpayer are not unduly complicated, even though the derivation of the tables that he uses may well remain a mystery to most. The tax may be made to vary in a reasonable manner, though some of the desirable properties are found on examination to be mutually incompatible. It is not essential that the tables used conform to the formula developed here; that exhibited in (60) is intended primarily as a demonstration that suitable formulas can be developed. A table could presumably be set up without reference to any analytic function, although the use of such a function would make it easier to ascertain that no undesired anomalies were present.

The enactment of such a tax law should have obvious advantages. Testators may dispose of their wealth in the way they want without having to consider capricious differences in tax resulting from minor differences in the disposition of their estate; questions of the precise occasion at which wealth is considered to be transferred will no longer be important, and the problem of evaluating contingent interests may be resolved by waiting until the interests vest irrevocably; greater equity should produce more willing acceptance of the tax and better compliance on the part of the taxpayer; and substantial increases in the supply of risk capital and decreases in the institutionalizing of investment should result.

Yet a shift to a tax of this sort will not be easy to obtain. In particular it may be difficult to obtain congressional approval of a mathematical formula, or even of a set of tables that is necessarily less directly related to the taxes to be paid than the ordinary surtax schedule. And in addition to the normal inertia against adopting anything so novel as this proposal, there will be a considerable vested interest of taxpayers, tax lawyers, and trust companies in the various loopholes of existing law. However, this scheme differs from the usual loophole-plugging patchwork in that it does not penalize innocent taxpayers or require them to take precautions lest a heavy tax be inadvertently incurred through a relatively trivial provision, but leaves the taxpayer free to select without penalty any method of transmission that meets his needs. For this reason it may well merit the support of even the taxpayer.

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