

CHAPTER IV.

ON THE DISTINCTION BETWEEN THE ABSTRACT AND THE INDUCTIVE SCIENCES, AND CLASSIFICATION OF THE ABSTRACT SCIENCES.

“A science is a complement of cognitions, having in point of form the character of logical perfection, and in point of matter the character of real truth.”*

ON THE DISTINCTION BETWEEN THE ABSTRACT AND THE INDUCTIVE SCIENCES.

[THE mathematical sciences are called *abstract*, the physical and psychological sciences are called *inductive*.]

1. Every science has an *object noun*, the relations of whose various forms are the subjects of investigation.

This object noun is the first necessary condition of the existence of science.

2. When the object noun is known or appreciated

* This beautiful definition was given by Sir William Hamilton in his course of lectures on Logic, delivered in the college of Edinburgh.

by the mind, the mind considers the various forms of which it is susceptible, and classifies them.

3. When the classification has made a certain degree of progress, the mind can investigate the relations of those various forms, and by the aid of the axioms that exist intuitively in all men, can connect the propositions by the law of reason and consequent.

A science is more or less perfect according to the number of propositions that are thus connected by the law of reason and consequent.

No number of isolated propositions can constitute a science. It is absolutely necessary that they be connected by the law of reason and consequent; that is, that one proposition (called technically the consequent) shall result from two other propositions, called the premises.

To illustrate these propositions let us turn our attention for a moment to geometry.

1st, The object noun of geometry is *space*.

2d, The mind first inquires into the various forms of which space is susceptible. These are determined and classed,—namely, into lines and figures; the lines into straight lines and curves; the figures into triangles, the square, the circle, &c.

3d, When the classification of these *forms of space* has made some progress, their relations may be investigated by the aid of the axioms that exist in the minds of all men. But propositions can be connected through axioms only by making one proposition a major premiss; another, a minor premiss; and the

third that results from them a consequent. Logic is therefore universally present in every *science*, but it is not present in the classification. The descriptive sciences (descriptive botany, descriptive anatomy, &c.) are not *sciences*, they are merely classifications. Their propositions are not connected by the law of reason and consequent, and thus every proposition is isolated from its fellow.

In every science, therefore, there is a major proposition, a minor proposition, and a consequent.

The essential difference between the abstract and the inductive sciences is this: In the abstract sciences we commence our reasonings with the major and minor premiss, and descend to the consequent; in the inductive sciences we commence our reasonings with the consequent and minor premiss, and ascend to the major premiss.

In every reasoning whatever, two propositions must be given to find the one that is required.

In the abstract sciences the propositions given are the major premiss (the *axiom*) and the minor premiss (the form of the object noun expressed in its description), and the proposition required is the necessary consequent that results from these two.

In the inductive sciences the propositions given are the consequent (namely, the observed phenomenon considered as *an effect*), the minor premiss (namely, the *conditions* of matter), and the required proposition is the major premiss,—namely, any such general proposition as will complete the syllogism,

and make the consequent follow necessarily from it and the minor premiss.

Viewed in this light, the necessity of *observation* in the inductive sciences, and its non-necessity in the abstract sciences, become immediately apparent.

Let it be remembered, that in every reasoning whatever, two propositions (two relations) must be given before we can discover the relation that is required.

Now, if any two propositions of a syllogism be given, the third follows necessarily and can be discovered.

In the abstract sciences, if we have *given* the forms of the object noun, the mind from its constitution supplies the major premiss, the axiom or general principle, and the consequent can be discovered without the aid of any external observation whatever.

But in the inductive sciences the general principle is the proposition required; and we can only arrive at it through the medium of the two other propositions of the syllogism, both of which must be obtained through observation, as one is the phenomenon, the other the actual condition of matter.

The difference, then, between the abstract and the inductive sciences is this: In the abstract sciences let the object noun be *given*, and the mind can investigate the relations of its various forms; in a word, can discover the *science* of that object noun. In the inductive sciences, on the contrary, it is not sufficient that the minor premiss (the conditions of matter) be given, the phenomena must also be *observed*, because

two propositions must always be given before the third can be discovered through reasoning, and the major or general principle of the inductive sciences does not exist in the human mind as does the major proposition of the abstract sciences.

OF THE ABSTRACT SCIENCES.

Each abstract science has an object noun, of the relations of whose forms that science exclusively treats.

The abstract sciences are Logic, Arithmetic, Algebra, Geometry, Statics, and Ethics.

The object noun of Logic is Identity.

The object noun of Arithmetic is Number.

The object noun of Algebra is Quantity.

The object noun of Geometry is Space.

The object noun of Statics is Force.

The object noun of Ethics is Equity.

The forms of Identity are terms singular, partial, or universal (considered as identical with or equal to each other in any particular syllogism).

The forms of Number are units, tens, hundreds, &c.

The forms of Quantity are x , $-x$, \sqrt{x} , x^2 , x^3 , &c.

The forms of Space are lines, angles, and figures.

The forms of Force are forces of particular quantity acting in particular directions; and the object of these various sciences is to discover the necessary and universal *relations* of these various forms.

It will be observed that we have given the sciences in a certain order. This order is not arbitrary, but the order in which they must necessarily be studied,

and the order in which they must necessarily be discovered.

Logic is the first and simplest of all the abstract sciences, and it is so simple that it need scarcely be studied objectively for the purpose of reasoning, although every advantage attends its study for the purpose of teaching us how we do reason.

Arithmetic is nothing more than logic applied to number.

Algebra is logic and arithmetic applied to quantity.

Geometry (in its larger sense) is logic, arithmetic, and algebra (the science of *quantity*) applied to space.

Statics is logic, arithmetic, algebra, and geometry applied to force.

The following table may be given of the abstract sciences:—

[ONTOLOGICAL, OR NATURAL.]				
Identity,				Logic.
”	Number,			Arithmetic.
”	”	Quantity,		Algebra.
”	”	”	Space,	Geometry.
”	”	”	”	Force, Statics.
[DEONTOLOGICAL, OR MORAL.]				
Equity,				Politics or Ethics.*

* The dogma of ethics must be as carefully distinguished from the human facts to which it is applied, as the dogma of mathematics and dynamics is from the physical facts to which it is applied in astronomy. Without the mathematical dogma, astronomy is impossible as a science. It is and can be only a superstition, and always was so till the dogma was introduced. So also practical politics, law, legislation; and the whole device of government and rule is only a great superstition until the ethical dogma (which must be first perfected) is introduced. Unless we distinguish between the abstract dogma and the real facts we can make no progress.

The object of the first five sciences is to inquire *what is*; the object of the last, to inquire *what ought to be done*. Ethics is distinguished by the name of a moral science, and is nothing more than logic applied to equity.

It will be observed that although the abstract sciences may be studied in their abstract form, they are of no possible use (objectively) until they are applied to the real objects that exist in the world; and also, it will be observed that *abstract* number, quantity, space, &c., have no existence whatever except in the mind, and that every *real* number, quantity, &c., is number of something, quantity of something, &c.

But not only is the existence of the subject represented by the object noun necessary to an abstract science, there must also be the axiom that forms the fundamental canon of the science. This canon in logic is, "Things that are identical with the same are identical with each other," which sometimes assumes the technical form of "whatever is predicated of a term may be predicated of that term's equivalent."* In arithmetic we have "the whole is equal to the sum of all its parts," which is the essential foundation of addition, and consequently of subtraction, the reverse of addition. But multiplication is only the addition of similar sums, and division is only demultiplication, consequently this canon is the essential basis of the four primary rules of arithmetic.

* The first form is the real canon of the *science*, the second the canon of the *language*; but the empirical logicians generally prefer the latter, as they seem determined that logic shall not be a science.

In algebra the canon is, "Equals added to equals give equals," which, of course, extends to subtraction, multiplication, and division, as those operations are only forms of addition, or of de-addition (that is, of addition reversed).

The canon of arithmetic is only a particular form of the canon of logic; the canon of algebra is only a particular form of the canon of arithmetic; so that logic presents the abstract form, arithmetic the abstract form applied to *number*, and algebra the abstract form (modified by arithmetic) applied to *quantity*.

These canons or axioms may or may not be *announced*; if not, they are always taken for granted as portions of the mental constitution.* Practically,

* "If all men had logical heads, metaphysic dogmas might be regarded as matters of indifference. They work mischief only in confused brains, just as the flaring comet, while coolly gazed at by the astronomer, who knows what he is looking at," (?) "actually stirs revolutions and overthrows empires, among superstitious nations."—(*Isaac Taylor*.) When we say that it is a matter of indifference whether axioms are announced or not, we speak exclusively of *science*, and not of *philosophy*. Each science takes *one single portion* of the mass, and investigates that *alone*, without ever going beyond the boundary that separates it from all the other portions. Were all men to do nothing but investigate geometric truth, they never would go beyond *space and its relations*. *Every* noun substantive they made use of would have a relation to the identity, equality, number, or quantity of the various spaces whose relations they investigated. Beyond this they could never go, were they to think for ever. The instrument that thinks (the mind, brain, or whatever it may be) has no more right to be the subject of their speculations as geometers, than the anatomy of the hand by which they write their signs. Philosophy, however, endeavours to grasp the whole series of phenomena contained within the whole region of our consciousness, and to class man and man's mind, man's abstract thought, and man's *axiom*, and to furnish a reasonable scheme by which things shall appear to be *correlative* to each other.

it is perhaps a matter of indifference whether they are announced or not; but there seems to be an advantage in stating them, as a science involves a rational *operation* (namely, the operation of proof), and as the axiom is the ultimate form of a proposition—the *most* abstract form in which it can be presented—a science is formally complete when the axiom is stated, and not otherwise.

The conditions, then, that permit of the existence of an abstract science are:—

1st, The existence of an object noun, abstract, simple, and incapable of definition.

2d, The possibility of stating the forms of that noun in primary propositions, called definitions.

3d, The existence of axioms applicable to the forms of the object noun.

If these conditions are fulfilled, an abstract science is possible; if they are not fulfilled (tacitly or explicitly), an abstract science is impossible.

We maintain that the abstract science of *equity* fulfils all these conditions, and it is the purpose of the following pages to attempt a demonstration of one or two of the more general and more simple propositions of the science.

[To avoid a complication of language (by the repeated limitation of certain terms to the abstract sciences), it may be as well to premise that *proof* is of two kinds; and when we speak of proofs in the science of equity, we speak of deductive, and not of inductive proof. In deductive proof the definition and the axiom represent the whole science, and every

proposition, to be correct, must be a particular case of them. Demonstration is only the exhibition of the process by which a proposition, whose truth is not self-evident, is shown to be *equivalent* to, or involved in, another proposition, whose truth is self-evident. Inductive proof, on the contrary, rests on an appeal to facts which can be *observed*; and observation never gives us the major proposition of a syllogism, but the minor and conclusion, from which the major is inferred. The reader will therefore bear in mind that when we speak of proof, or of propositions, we mean *in the abstract sciences*.]

It has usually been maintained that moral science is *not capable* of being reduced to the same accurate form as the other sciences. This assumption (and it is nothing more) is not peculiar to morals, but has also been applied to the physical sciences. *Every* branch of knowledge not yet reduced to scientific form is obnoxious to a similar objection; and there are excellent men who doubt the propriety of attempting to fathom the unknown,—forgetting that our fathers have been guilty of the same offence, and that we are quietly reaping the fruits of their labours. If equity be incapable of reduction to scientific form, it must have some distinguishing characteristic that places it on a different footing from the abstract sciences already reduced to systematic ordination.

But if the *conditions* of a science are fulfilled, all that remains to be done is, to apply the intellect, and as a *simple* concept (one that is incapable of

definition) is just as simple whatever be its nature, it is perfectly absurd to object to the application of logic to that concept, because it happens to be the one that men have not yet specifically investigated.

The science of equity is less complex than geometry, and much less complex than mechanics; and although men have not been in the habit of viewing politics in this light, we must not forget that the generality of mankind are wedded to their accustomed course of thought, and believe nothing possible until it is actually realised. At the same time, equity is pretty generally admitted to *exist*; all *law* being supposed to be founded on equity, and to derive its authority, not from the fact of its *enactment*, but from its *justice*. Enactment is only the *publication* or the *application* of the rules of equity. Now, equity either has rules or it has no rules. If it have rules, let them be investigated, on the same principle as the rules of arithmetical or geometric calculation. If it have no rules, let us abandon the subject altogether, and let there be no right and no wrong, and let every one do as he pleases.

The possibility of reducing equity to a science admits of only one answer that is of any practical importance. Men will admit the science when it *is* a science, and not till then.