Consumption Versus Wage Taxation

Author(s): Elhanan Helpman and Efraim Sadka

Source: *The Quarterly Journal of Economics*, May, 1982, Vol. 97, No. 2 (May, 1982), pp. 363-372

Published by: Oxford University Press

Stable URL: https://www.jstor.org/stable/1880764

CONSUMPTION VERSUS WAGE TAXATION*

ELHANAN HELPMAN AND EFRAIM SADKA

I. INTRODUCTION

In the commonly used model in the tax literature, there is a single factor of production, labor, and production takes place with constant returns to scale. As a result, there are no profits in a competitive equilibrium, and net wage income equals expenditure on consumption goods. It is well-known that in this case a wage tax is equivalent to a *uniform* tax on all consumption goods. Hence, in the absence of pure profits, one can always do as well with *uniform* consumption taxation (indirect taxation) as with a wage tax (direct taxation). If it is further allowed to levy taxes at different rates on consumption goods, then one can expect to do better with commodity taxes than with a wage tax. Nevertheless, this is not always true: there are cases in which a wage tax is as good as any system of excise taxes on consumption goods (see Diamond and Mirrlees [1971], Atkinson and Stiglitz [1972], Sandmo [1974], and Sadka [1977]).

In this paper, we examine this result for economies possessing decreasing-returns-to-scale technologies and show that in such economies a wage tax is always inferior to consumption taxation (see Section III). In a preliminary section (Section II) we present the model and prove a simple version of a result obtained by Foster and Sonnenschein [1970], Kawamata [1974], and Rader [1976], concerning the welfare effects of radial changes in price distortions.

II. PRELIMINARY RESULTS

Let the economy consist of one consumer (or, alternatively, many identical consumers) who consumes n goods. His consumption set C is R^n_+, and his initial endowment is $w = (w_1, \ldots, w_n) \in R^n_+$.

ASSUMPTION 1. The consumer's preferences can be represented by a quasi-concave, continuously differentiable utility function (u), which is defined on R^n_+ and has positive first-order partial derivatives.

The government's consumption takes the form of a fixed com-

* Financial support from the Foerder Institute for Economic Research is gratefully acknowledged.

The Quarterly Journal of Economics, May 1982 CCC 0033-5533/82/020363-10$02.00

modity vector $z > 0$.[1] The aggregate production set is denoted by $Y \subset R^n$, and the efficient production frontier by

$$\Pi = \{y \in Y / y' \notin Y \quad \text{for all } y' > y\}.$$

ASSUMPTION 2. (a) Y is closed and convex and admits free disposal; (b) there exists a continuously differentiable real-valued function F: $R^n \to R$ such that the efficient production frontier Π can be expressed as $\Pi = \{y \in Y/F(y) = 0\}$; and (c) F has positive first-order partial derivatives.[2]

The set $(Y + \{w\} - \{z\}) \cap R^n_+$ consists of the private consumption bundles that are technologically feasible, after providing for the government's consumption. It is called the attainable set.

ASSUMPTION 3. The attainable set $(Y + \{w\} - \{z\}) \cap R^N_+$ is non-empty and compact.[3]

Following Foster and Sonnenschein [1970], Kawamata [1974], and Rader [1976], we define an equilibrium with a specific price distortion vector, $d = (d_2, d_3, \ldots, d_n) \in R^{n-1}$ or, in short, a d-equilibrium as follows:

DEFINITION I. Let $d \in R^{n-1}$. An allocation (x,y) is a d-equilibrium if (i) $x \in R_n^+$; (ii) $y \in \Pi$; (iii) $x = y + w - z$; and (iv) $u_i(x)/u_1(x) - F_i(y)/F_1(y) = d_i$ for $i = 2,3, \ldots, n$.[4]

Generally, there may be more than one equilibrium associated with a given price distortion vector d—in which case there may be more than one equilibrium utility level associated with this distortion vector. If this is the case, welfare comparisons between two distortion vectors are ambiguous. In order to avoid this difficulty, we make the following assumption.

ASSUMPTION 4. (a) For every $d \in R^{n-1}$, there exists at most one d-equilibrium, denoted by $[x(d),y(d)]$. (b) If a d-equilibrium exists, then for every $0 \leqq \theta \leqq 1$, there exists a $\theta\, d$-equilibrium.

LEMMA 1. $x(d)$ and $y(d)$ are continuous in d.

Proof. Notice that $x(d) = y(d) + w - z$. Thus, a point of dis-

1. For $x,y \in R^n$ with $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$: $x \geqq y$ means $x_i \geqq y_i$ for all i; $x > y$ means $x_i \geqq y_i$ for all i and $x \neq y$; $x \gg y$ means $x_i > y_i$ for all i.
2. Since F is continuous, it follows that Π is closed.
3. The compactness of the attainable set follows from Assumption 2 if we further assume that it is impossible to produce "something from nothing," namely, $Y \cap R^n_+ \subset \{0\}$ (see Arrow and Hahn [1971]).
4. By u_i we denote the partial derivative of u with respect to the ith argument, and similarly for F_j.

continuity of $x(d)$ is also a point of discontinuity of $y(d)$ and vice versa. Suppose, contrary to the assertion of the lemma, that there exists $d^0 \in R^{n-1}$, which is a point of discontinuity of $x(d)$. Since for all d, $x(d) \in (Y + \{w\} - \{z\}) \cap R^n_+$, which is a compact set, it follows that there is a sequence $\{d^m\}$ in R^{n-1} that converges to d^0 and such that $\{x(d^m)\}$ converges to some point $\bar{x} \in R^n_+$, which is different from $x(d^0)$. Since $y(d^m) = (x(d^m) - w + z) \in \Pi$ and since $\Pi = \{y \in Y/F(y) = 0\}$ is closed, it follows that $\{y(d^m)\}$ converges to $\bar{y} = (\bar{x} - w + z) \in \Pi$. From the continuity of u_i and F_i, it follows that

$$\frac{u_i[x(d^m)]}{u_1[x(d^m)]} - \frac{F_i[y(d^m)]}{F_1[y(d^m)]} \rightarrow \frac{u_i(\bar{x})}{u_1(\bar{x})} - \frac{F_i(\bar{y})}{F_1(\bar{y})}, \quad i = 2, \ldots, n.$$

On the other hand, since $[x(d^m), y(d^m)]$ is a d^m-equilibrium, it follows that

$$\frac{u_i[x(d^m)]}{u_1[x(d^m)]} - \frac{F_i[y(d^m)]}{F_1[y(d^m)]} = d_i^m \rightarrow d_i^0, \quad i = 2, \ldots, n,$$

where $d^m = (d_2^m, \ldots, d_n^m)$ and $d^0 = (d_2^0, \ldots, d_n^0)$. Thus,

$$\frac{u_i(\bar{x})}{u_1(\bar{x})} - \frac{F_i(\bar{y})}{F_1(\bar{y})} = d_i^0, \quad i = 2, \ldots, n.$$

Hence, $(\bar{x}, \bar{y})$ is a d^0-equilibrium, and $(\bar{x}, \bar{y}) \neq (x(d^0), y(d^0))$. This contradicts Assumption 4(a). Q.E.D.

For each distortion $d \in R^{n-1}$, let $v(d)$ be the utility level associated with the distortion. $v(d)$ is an indirect utility function defined by

$$v(d) \stackrel{\text{def}}{=} u[x(d)].$$

Since u is continuous, the following is a corollary of Lemma 1.

COROLLARY 1. $v(d)$ is continuous in d.

The next lemma deals with radial decreases in distortions and follows from the works of Kawamata [1974] and Rader [1976]. For convenience we present a simple direct proof.

LEMMA 2. A radial decrease in distortions increases welfare. Formally, for every $d \neq 0$ and $0 \leqq \theta < 1$, $v(\theta d) > v(d)$.

Proof. It is straightforward to verify that $\mathbf{0}$ − equilibrium (i.e., a no-distortion equilibrium) is Pareto optimal. Hence, $v(0) \geqq v(d)$ for all d. Suppose that $d \neq 0$. If $v(\theta d) \leqq v(d)$, then there must exist $\bar{\theta}$ with $0 \leqq \bar{\theta} \leqq \theta < 1$, such that $v(\bar{\theta} d) = v(d)$ (by the continuity of v). We shall show that this is impossible.

Let (x^1,y^1) and (x^2,y^2) be the d-equilibrium and $\bar{\theta}d$-equilibrium, respectively. By supposition, $u(x^1) = u(x^2)$. Put,

$$q^i = \left(1, \frac{u_2(x^i)}{u_1(x^i)}, \ldots, \frac{u_n(x^i)}{u_1(x^i)}\right), \quad i = 1,2$$

$$p^i = \left(1, \frac{F_2(y^i)}{F_1(y^i)}, \ldots, \frac{F_n(y^i)}{F_1(y^i)}\right), \quad i = 1,2$$

$$t^1 = q^1 - p^1 = (0,d), \quad t^2 = q^2 - p^2 = (0,\bar{\theta}d).$$

Since, by definition, $t^2 = \bar{\theta}t^1$, $t^1 \neq 0$ and $\bar{\theta} < 1$, it follows that $t^1 \neq t^2$ and hence either $p^1 \neq p^2$ or $q^1 \neq q^2$ (or both). It follows from Assumption 2 that

(1) $\quad p^1y^1 \geq p^1y^2$ and $p^2y^1 \leq p^2y^2$,

with a strict inequality holding if $p^1 \neq p^2$. Similarly, Assumption 1 implies that

(2) $$q^1x^1 \leqq q^1x^2 \quad \text{and} \quad q^2x^1 \geqq q^2x^2,$$

with a strict inequality holding if $q^1 \neq q^2$. Recalling that either $p^1 \neq p^2$ or $q^1 \neq q^2$, it follows from (1) and (2) that

(3) $\quad q^1x^1 - p^1y^1 < q^1x^2 - p^1y^2$ and $q^2x^1 - p^2y^1 > q^2x^2 - p^2y^2$.

Since, by definition, $x^i = y^i + w - z$, $i = 1,2$, it follows from (3) that

(4) $$t^1y^1 < t^1y^2 \text{ and } t^2y^1 > t^2y^2.$$

Since $t^2 = \bar{\theta}t^1$, it follows from (4) that

$$t^1y^1 < t^1y^2 \quad \text{and} \quad \bar{\theta}t^1y^1 > \bar{\theta}t^1y^2.$$

This is a contradiction because $\bar{\theta} \geqq 0$. Q.E.D.

III. Wage Taxation Versus Consumption Taxation

In this section we compare two types of taxation: a wage tax ("direct taxation") and taxes on consumption goods ("indirect taxation"). For this purpose we define a competitive equilibrium with commodity taxes (or subsidies). This equilibrium is characterized by two price vectors: one for consumers and one for producers. The difference between these price vectors is the vector of specific excise taxes.

Definition II. Let $q, p \in R^n$ with $q, p \gg 0$. An allocation (x,y) is a (q,p)-equilibrium if

(i) $$x = y + w - z;$$

(ii) $$x \text{ maximizes } u(x') \text{ on } \{x' \in R^n_+ / qx' \leqq qx\};$$

(iii) $$y \text{ maximizes } py' \text{ on } Y \text{ (which implies that } y \in \Pi).$$

In this definition, q serves as a consumer price vector, and p as a producer price vector. The vector $t = q - p$ is viewed as an excise tax vector. However, t is not necessarily the only form of taxation that prevails in a (q,p)-equilibrium. There may exist also a lump-sum tax or subsidy. To see this, observe that the consumer's income (from the sale of his initial endowment and the profits of the firms that he owns) is $qw + py$, while his expenditure is qx. The difference between these two values $T = qw + py - qx$ is a lump-sum tax (which could also be negative). This motivates the next definition:

DEFINITION III. Let $q, p \in R^n$ with $q,p \gg 0$. An allocation (x,y) is a (q,p)-equilibrium without lump-sum taxation (in short, a (q,p)-equilibrium WOLST) if

(i) $$(x,y) \text{ is a } (q,p)\text{-equilibrium;}$$

(ii) $$qx = qw + py.$$ [5]

Both of these concepts of equilibria are referred to as tax-equilibria.

Notice that if (x,y) is a (p,p)-equilibrium, then utility maximization (condition (ii)) implies that $q_i/q_1 = u_i(x)/u_1(x)$, with $i = 2, \ldots, n$. Similarly, profit maximization (condition (iii)) implies that $p_i/p_1 = F_i(y)/F_1(y)$, $i = 2, \ldots, n$. Define

$$d = \left(\frac{q_2}{q_1} - \frac{p_2}{p_1}, \ldots, \frac{q_n}{q_1} - \frac{p_n}{p_1}\right). \tag{5}$$

Then (x,y) is also a d-equilibrium, where d is defined in (5). Conversely, let (x,y) be a d-equilibrium and define

$$q = \left(1, \frac{u_2(x)}{u_1(x)}, \ldots, \frac{u_n(x)}{u_1(x)}\right) \tag{6}$$

and

$$p = \left(1, \frac{F_2(y)}{F_1(y)}, \ldots, \frac{F_n(y)}{F_1(y)}\right) \tag{7}$$

(and hence $q - p = (0,d)$). We shall show that in this case (x,y) is a

5. Notice that our definition of a (q,p)-equilibrium WOLST allows for a pure-profit tax or subsidy. Specifically, if (x,y) is a (q,p)-equilibrium WOLST, then (x,y) is also a $(q,(1 + \lambda)p)$-equilibrium WOLST but with a pure-profit tax rate of $1 - 1/(1 + \lambda)(\lambda > -1)$, where $\lambda > -1$.

(q,p)-equilibrium. Conditions (i) and (iii) of Definition II trivially hold. But instead of condition (ii) we have

(iv) $\quad x$ minimizes qx' on $\{x' \in R^n_+ / u(x') \geqq u(x)\}$.

In other words, instead of utility maximization we have expenditure minimization. However, recall that $q \gg 0$ (see (6)). Hence, if $x > 0$, there must exist $\bar{x} \in R^n_+$, which costs strictly less than x (i.e., $q\bar{x} < qx$). Thus, (ii) follows from (iv) (see Debreu [1959]). If $x = 0$, then x is the only bundle of R^n_+, that costs $qx = 0$, in which case (ii) trivially holds. Thus, we have proved

LEMMA 3. If (x,y) is a (q,p)-equilibrium, then it is also a d-equilibrium, where d is defined by (5). Conversely, if (x,y) is a d-equilibrium, then it is also a (q,p)-equilibrium, where q and p are defined by (6) and (7), respectively.

We are now in a position to show that there exist taxes on consumption goods that are strictly preferred (from the consumer's welfare point of view) to a wage tax (both systems financing the government's vector of spending z), if in the equilibrium with the wage tax there is a positive aggregate pure-profit. We let good 1 be leisure and the other $n - 1$ goods be consumption goods. In order to understand Theorem 1 (below), notice that a wage tax prevails whenever the consumer price of leisure is lower than the producer price. Consumption taxes or subsidies exist whenever the consumer prices of the consumption goods differ from the producer prices.

THEOREM 1. Let $q, p \in R^n$ with $q = (q_1, \ldots, q_n)$, $p = (p_1, \ldots, p_n)$, $q,p \gg 0$, $q_1 < p_1$ and $q_i = p_i$ for $i = 2, \ldots, n$. Suppose that (x,y) is a (q,p)-equilibrium WOLST and that $py > 0$. (In other words, (x,y) is a tax equilibrium with only one form of taxation: direct (wage) taxation.) Then there exist $\bar{q}, \bar{p} \in R^n$, with $\bar{q} = (\bar{q}_1, \ldots, \bar{q}_n) \gg 0$ and $\bar{p} = (\bar{p}_1, \ldots, \bar{p}_n) \gg 0$, and $\bar{x},\bar{y} \in R^n$ such that

(i) $$\bar{q}_1 = \bar{p}_1;$$

(ii) $(\bar{x},\bar{y})$ is a $(\bar{q},\bar{p})$-equilibrium WOLST;

(iii) $$u(\bar{x}) > u(x).$$

(In other words, $(\bar{x},\bar{y})$ is a tax equilibrium with only indirect (consumption) taxation that is strictly preferred to the wage tax equilibrium (x,y).)

Proof. Define $\hat{p} = (1, \hat{p}_2, \ldots, \hat{p}_n) = (1/p_1)\,p$ and $\hat{q} = (1, \hat{q}_2, \ldots, \hat{q}_n) = (1/q_1)q$. Since $qx = qw + py$ (by (ii) of Definition III), it follows that $(1/q_1)(qx) = (1/q_1)(qw) + (1/q_1)(py)$. Since $q_1 < p_1$ and $py > 0$, we must have $(1/q_1)(qx) > (1/q_1)(qw) + (1/p_1)(py)$, or, equivalently,

$$\hat{q}x > \hat{q}w + \hat{p}y. \tag{8}$$

Since (x,y) is a (q,p)-equilibrium, it follows that $u_i(x)/u_1(x) = q_i/q_1 = \hat{q}_i, i = 2, \ldots, n$, and $F_i(y)/F_1(y) = p_i/p_1 = \hat{p}_i, i - 2, \ldots, n$. We can then conclude from (8) that

$$\left(1, \frac{u_2(x)}{u_1(x)}, \ldots, \frac{u_n(x)}{u_1(x)}\right)x > \left(1, \frac{u_2(x)}{u_1(x)}, \ldots, \frac{u_n(x)}{u_1(x)}\right)w + \left(1, \frac{F_2(y)}{F_1(y)}, \ldots, \frac{F_n(y)}{F_1(y)}\right)y. \tag{9}$$

Since (x,y) is a (q,p)-equilibrium, it follows from Lemma 3 that it is a $\hat{d}$-equilibrium, where $\hat{d} = (q_2/q_1 - \hat{p}_2/p_1, \ldots, q_n/q_1 - p_n/p_1) = (\hat{q}_2 - \hat{p}_2, \ldots, \hat{q}_n - \hat{p}_n)$. Let (x^0,y^0) be a $\mathbf{0}$-equilibrium. It is also a (p^0,q^0)-equilibrium for some $q^0,p^0 \in R^n$ with $q^0 = p^0$. Since $x^0 = y^0 + w - z$, it follows that $q^0x^0 = p^0y^0 + q^0w - p^0z$ and hence (recall that $z > 0$)

$$q^0x^0 < q^0w + p^0y^0.$$

Thus,

$$\left(1, \frac{u_2(x^0)}{u_1(x^0)}, \ldots, \frac{u_n(x^0)}{u_1(x^0)}\right)x^0 < \left(\frac{u_2(x^0)}{u_1(x^0)}, \ldots, \frac{u_n(x^0)}{u_1(x^0)}\right)w + \left(1, \frac{F_2(y^0)}{F_1(y^0)}, \ldots, \frac{F_n(y^0)}{F_1(y^0)}\right)y^0. \tag{10}$$

Comparing (9) and (10), it follows from the continuity of u_i,F_j, and the d-equilibrium (Lemma 1), that there exists $0 < \bar{\theta} < 1$ such that the $\bar{\theta}\hat{d}$-equilibrium (denote it by $(\bar{x},\bar{y})$) satisfies

$$\left(1, \frac{u_2(\bar{x})}{u_1(\bar{x})}, \ldots, \frac{u_n(\bar{x})}{u_1(\bar{x})}\right)\bar{x} = \left(1, \frac{u_2(\bar{x})}{u_1(\bar{x})}, \ldots, \frac{u_n(\bar{x})}{u_1(\bar{x})}\right)w + \left(1, \frac{F_2(\bar{y})}{F_1(\bar{y})}, \ldots, \frac{F_n(\bar{y})}{F_1(\bar{y})}\right)\bar{y}. \tag{11}$$

By Lemma 2, $v(\bar{\theta}\hat{d}) > v(\hat{d})$, namely, $u(\bar{x}) > u(x)$. Define

$$\bar{q} = \left(1, \frac{u_2(\bar{x})}{u_1(\bar{x})}, \ldots, \frac{u_n(\bar{x})}{u_1(\bar{x})}\right) \text{ and } \bar{p} = \left(1, \frac{F_2(\bar{y})}{F_1(\bar{y})}, \ldots, \frac{F_n(\bar{y})}{F_1(\bar{y})}\right).$$

Then, $(\bar{x},\bar{y})$ is the required $(\bar{q},\bar{p})$-equilibrium WOLST (see (11)).

Q.E.D.

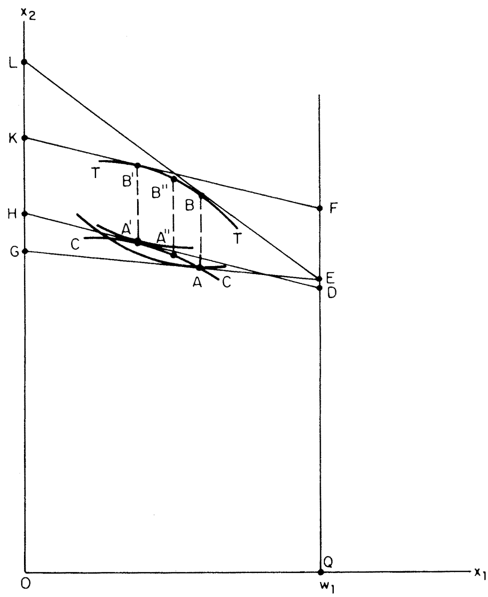

FIGURE I

To help the reader's intuition, we provide a heuristic explanation of our main result that is stated in Theorem 1. First, observe that a wage tax is equivalent to a uniform tax on consumption goods and a subsidy to pure profits. In order to see this, write the consumer's budget constraint in the presence of a wage tax as

$$(12)\quad (1-\tau)p_1x_1 + \sum_{i=2}^{n} p_ix_i = (1-\tau)p_1w_1 + \sum_{i=2}^{n} p_iw_i + py$$

where p is the vector of producer prices and τ is the tax rate on wage income. Dividing (12) by $1-\tau$, we obtain

$$(13)\quad p_1x_1 + \sum_{i=2}^{n} (1+\alpha)p_ix_i = p_1w_1 + \sum_{i=2}^{n} (1+\alpha)p_iw_i + (1+\alpha)py,$$

where $1 + \alpha = (1-\tau)^{-1}$. It is clear that (13) represents a budget constraint for the case in which there is a uniform tax rate on the consumption goods that is equal to α and a subsidy to profits at the same rate α.

Now specialize the model to the case of a single consumption good ($n = 2$), with $w_2 = 0$ and $z = (0,z_2)$. Points A and B in Figure I represent the equilibrium consumption and production vectors, respectively, for the wage tax equilibrium. The curve TT represents a portion of $\Pi + w$, the production frontier (Π translated by the initial endowment vector w). The curve CC represents a portion of the consumption possibility frontier that is $\Pi + w - z$. The vertical distance between TT and CC is z_2—the government's real consumption. Production plus initial endowment is at B, generating profits whose real value in terms of the consumption good is $EQ = py/p_2$. Due to the wage tax, the consumer's budget line EG has a lower slope than the producer's iso-profit line EL. In fact, $|\text{slope of } EG| = (1 - \tau)p_1/p_2 = (1 - \tau)|\text{slope of } EL|$. Now, in order to interpret this equilibrium as an equilibrium with a tax at rate α on the consumption good and a subsidy at the same rate α to profits, observe that nominal profits including the subsidy are $(1 + \alpha)py$. The real value of profits to the consumer is $(1 + \alpha)py/[(1 + \alpha)p_2] = py/p_2 = EQ$, as in the wage tax case. Also, due to the consumption tax, the slope of the consumer's budget line EG is $p_1/(1 + \alpha)p_2 = (1 - \tau)p_1/p_2$, which is $(1 - \tau)$ times the slope of the producer's iso-profit line EL, as in the wage tax case.

We present also in Figure I the first-best allocation. This is described by points A' and B'. Point A' is the consumption point that is characterized by a tangency of an indifference curve to the consumption possibility frontier CC. Point B' is the point of production plus initial endowment that is located at the intersection of TT with a vertical line through A'. The distance between B' and A' is, of course, z_2.

The first-best allocation can be obtained by means of decentralization with the aid of a lump-sum tax (and no other distortionary taxes). Alternatively, if z_2 is not too large, it can be obtained with the aid of a tax on profits. Suppose that this is indeed the case. Then line DH in Figure I can be considered as representing the individual's budget constraint in the decentralized economy with a profit tax at the rate DF/FQ. The line FK is tangent to TT at B', so that FQ represents real profits at producer prices (= consumer price), while DH is tangent to CC and the indifference curve at A'. The line DH is parallel to FK as there are no taxes on the consumption good nor on wage income.

Now suppose that we move from (A',B') to (A,B) along $B'B$ and $A'A$ by changing the tax rate on the consumption good and the profit tax so as to preserve equilibrium along the way. Notice that the consumption tax is zero at (A',B'), but eventually becomes positive (at the rate α) at (A,B). The profit tax is positive at (A',B'), but eventually becomes negative (a subsidy) at (A,B). Thus, the move from

(A',B') to (A,B) can be done by raising the tax on consumption and lowering the rate of tax on pure profits. By continuity considerations, there exists a point, say (A'',B''), somewhere along the way from (A',B') to (A,B), where the consumption tax is positive and the profit tax is zero. As can be seen in Figure I, the utility at A'' is higher than at A. This proves the superiority of consumption taxation (A'') over wage taxation (A).

TEL AVIV UNIVERSITY

REFERENCES

Arrow, K. J., and F. H. Hahn, *General Competitive Analysis* (San Francisco: Holden-Day; and Edinburgh: Oliver & Boyd, 1971).

Atkinson, A. B., and J. E. Stiglitz, "The Structure of Indirect Taxation and Economic Efficiency," *Journal of Public Economics* (1972), 97–119.

Debreu, G., *Theory of Value* (New York: Wiley, 1959).

Diamond, P. A., and J. A. Mirrlees, "Optimal Taxation and Public Production," *American Economic Review*, LXI (1971), 8–27 and 261–78.

Foster, E., and H. Sonnenschein, "Price Distortion and Economic Welfare," *Econometrica*, XXXVIII (1970), 281–97.

Kawamata, K., "Price Distortion and Potential Welfare," *Econometrica*, XLII (1974), 435–60.

Rader, T., "The Welfare Loss from Price Distortions," *Econometrica*, XLIV (1976), 1253–57.

Sadka, E., "A Theorem on Uniform Taxation," *Journal of Public Economics*, VII (1977), 387–91.

Sandmo, A., "A Note on the Structure of Indirect Taxation," *American Economic Review*, LXIV (1974), 701–06.