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A FORMAL MODEL OF THE ECONOMY

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Persecution is used in theology, not in arithmetic, because in arithmetic there is knowledge, but in theology there is only opinion. So whenever you find yourself getting angry about a difference of opinion, be on your guard; you will probably find, on examination, that your belief is getting beyond what the evidence warrants. [BERTRAND RUSSELL, *Unpopular Essays*]

I. INTRODUCTION AND METHODOLOGY

Econometric models are used for many purposes. The chosen purpose of a model dictates, to a large extent, both the method of constructing the model and the data required for estimating its parameters.

Unfortunately, models are sometimes applied to purposes other than those for which they were developed. Often, the model builder fails to specify the purposes clearly and confusion results.

The purpose of the model described in this paper is to develop statistical relationships which "explain" a number of key economic variables. In so doing, we are able to test a number of statistical hypotheses that arise from economic reasoning. However, it is worth emphasizing at the outset that we make no attempt to estimate the parameters of any a priori structural model. No claim is made that the direction of cause-and-effect relationships can be inferred from the numerical results. For this reason, the only forecasts that can be produced from this model are those which are conditional on the assumed ex post behavior of chosen variables.

In the same vein, macroeconomic data are employed for specific ends. Here, too, confusion has resulted because of a failure to specify clearly the purposes of these data. Some care is necessary in choosing the data that are appropriate to statistical hypothesis testing of the kind reported here.

A. PREADJUSTMENT VERSUS "COADJUSTMENT" OF THE DATA

When describing current conditions or making comparisons, it is not only reasonable, but highly commendable to adjust the original data in order to eliminate unusual or transitory aberrations. The more fundamental changes or differences are thereby displayed in greater prominence. Assuming that the adjustment procedures are well thought out and accurately implemented, it is eminently reasonable to correct time series data for recurrent seasonal fluctuations —fluctuations due to strikes and holidays, and so on—in order to illustrate

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cycles and trends.[1] The general public and most policy makers are not capable of eliminating these transient variations as effectively as are professional statisticians. Such adjusted data are also suitable for econometric models whose purpose is to simulate the implications of a priori hypotheses.

But for the purpose of conditional forecasting, and especially of hypothesis testing, seasonally adjusted data are inappropriate for several reasons. First, seasonal adjustments which are arrived at in part through smoothing the original data tend to remove some of the behavioral covariance in the guise of seasonality. Second, seasonal adjustments, because of their averaging aspects, may tend to introduce autocorrelation into the data. This averaging can also obfuscate the timing of statistical relationships. Third, hypothesis tests which use seasonally adjusted data do not take account of the inevitable loss of degrees of freedom. In fact, for many preadjustment procedures, it would be no easy task to estimate just how many degrees of freedom have been lost. In such cases, the precision of the model in which the data are used, as well as the significance of its parameters, could be seriously overestimated.

The case against using data which have been adjusted for seasonality (accurately or not) is impressive in the context of hypothesis testing.[2] In any case, preadjustment for seasonality is not necessary. Many adjustment procedures can be developed in a single set of calculations with the process of estimating the model ("coadjustment"). Intuitively the most appealing method of coadjustment is the use of dummy variables, the coefficients of which will reflect the net seasonality in the equation.[3] In this way, the number of degrees of freedom lost is minimized (equal to the number of dummies required). Moreover, the statistical significance of the net seasonality can itself be examined directly. If one is concerned that the parameters themselves may vary seasonally or that the pattern of seasonality may change over time, these possibilities too can be tested by introducing additional dummies into the equation.[4]

The amount of information lost in the process of preadjustment may be small. It may, however, be substantial. The actual amount of information lost depends upon the specific situation. We do not know a priori how great the differences in the estimates will be.

Although these issues are well known to statisticians, virtually every quarterly macroeconomic model to date has been estimated using preadjusted data.[5] These

[1] Although the accuracy of seasonal adjustment procedures is not the concern of this paper, it is interesting to note a paper, "Some Problems in Estimating Short-Term Changes in GNP," by Rosanne Cole, prepared for the Allied Social Science Association meetings on December 27–30, 1970.

[2] See Lawrence Klein, *An Introduction to Econometrics* (New York: Prentice-Hall, Inc., 1962), p. 35; E. Malinvaud, *Statistical Methods of Econometrics* (New York: Rand McNally & Co., 1966), pp. 402–5; Michael Lovell, "Alternative Axiomatizations of Seasonal Adjustment," *Journal of the American Statistical Association* 61 (September 1966): 800–802; and George Ladd, "Regression Analysis of Seasonal Data," *Journal of the American Statistical Association* 59 (June 1964): 402–21.

[3] In the context of applying least squares to linear equations, it would be most convenient for computational purposes to use additive dummies. However, there is no requirement that any such specific functional form be adopted.

[4] See Lawrence Klein, *A Textbook of Econometrics* (New York: Row, Peterson, 1953), p. 316; Michael Lovell, "Seasoned Adjustment of Economic Time Series and Multiple Regression Analysis," *Journal of the American Statistical Association* 58 (December 1963): 993–1010.

[5] For an exception, see Lawrence Klein, R. J. Ball, A. Hazlewood and P. Vendome, *An Econometric Model of the United Kingdom* (Oxford: Blackwell, 1961), esp. chap. 3.

models are currently being used for hypothesis testing and forecasting purposes in addition to their legitimate purpose of simulation. In view of the above problems, the published results must be treated cautiously.

In this paper, the original data have been coadjusted using dummy variables in the course of estimating the model developed below. The purpose of the exercise has been to construct a set of empirical relationships which describe the postwar United States experience.

B. THEORETICAL UNDERPINNINGS

The three basic theoretical constructs represented in our equations are the traditional Keynesian, Quantity Theory, and Efficient Markets points of view. Within the Keynesian framework, budget outlays are allowed to compete with two more traditionally acceptable variables, namely, government purchases of goods and services and federal tax receipts. Within the Quantity Theory framework, the money supply plays the prime role. As many adherents of either the Quantity Theory or the Keynesian positions would find appropriate, the rate of unemployment is included to allow for the difference between price and real output responses to economic stimuli.

Efficient Markets theory holds that, at any moment in time, all market transaction prices reflect the best currently available information.[6] All extraordinary anticipated profit opportunities are presumed to be bid away by private interests. This theory is the very essence of profit maximization. One implication is that the current market value of all equities represents, in part, the present value of unbiased efficient forecasts of future economic returns. Another implication is that interest rates over the time horizon for which they apply, in part, reflect an unbiased efficient forecast of the future rate of inflation over the same time horizon.[7] To the extent that market participants possess information about the future, adherents of the Efficient Markets position hold that the value of this information will already have been incorporated into stock prices and interest rates as well as other prices.

C. THE EQUATIONS

With respect to the development of the Office of Management and Budget (OMB) model, the data are left as free as possible to tell their own story. Many of the models in the literature are constructed subject to constraints which represent the a priori theoretical beliefs of the model builders. They possess some of the characteristics of simulations as well as the characteristics of hypothesis tests. In other words, inferences drawn from the results are conditional on built-in assumptions as to the "true" structural relationships. The OMB model does not fall into this category. Constraints on the form and nature of the equations have been held to the minimum necessary to allow construction of explicit relationships.

The model consists essentially of three distinct equations. The first and second relationships are the nominal income-growth equation and a GNP-deflator equation, respectively. The third relationship deals with the rate of change of the rate of unemployment. When combined, the first two equations can be used

[6] For a review of some of this literature, see Eugene Fama, "Efficient Capital Markets: A Review of Theory and Empirical Work," *Journal of Finance* 25 (May 1970): 383–417.

[7] See Arthur Laffer and Richard Zecher, "Anticipations and Changes in the Value of Money—Much Ado about Nothing," mimeographed (University of Chicago, August 1970).

to provide conditional forecasts of real GNP. These forecasts can then be used in the third equation to develop conditional forecasts of changes in unemployment.

The following section describes the formal properties of the model; the third and final section summarizes the findings.

II. THE FORMAL MODEL

One purpose of a formal model of the kind reported here is to display in a systematic manner the statistical relationships among different economic variables. Although most of the attention is focused on those associations which are economically meaningful and statistically significant, some of the more interesting observations pertain to relationships which are widely alleged to be significant, but which, it turns out (at least within this context), are not.

A formal model can also provide statistical "explanations" for certain key variables. Such "explanations" may then be used to obtain estimates for these variables, given the values of a limited number of other variables. The key variables for which conditional forecasts could be made by means of this model are nominal GNP growth, the rate of change of the GNP deflator, real GNP growth, and the rate of change of the unemployment rate. These variables are, of course, among the most important economic magnitudes which concern policy makers.

A. SELECTION OF RELATIONSHIPS, VARIABLES, AND ESTIMATION PERIODS

By definition, the growth rate of nominal GNP less the rate of change of the GNP deflator equals the growth rate of real GNP.[8] Because of this identity, it is possible for two separate equations to provide an "explanation" for three variables—nominal GNP growth, the rate of change of the GNP deflator, and real GNP growth. We chose to concentrate our efforts on nominal GNP growth and the rate of change of the GNP deflator (equations [1] and [2], respectively). The results of internally consistent estimates should be independent of which two variables happen to be selected. As it turns out, the results are virtually independent of the decision to omit real GNP growth from direct estimation (equation [4] vis-à-vis equation [1] minus [2]).

The time intervals used in estimating each of the equations ended with the fourth quarter of 1969. They began with the first quarter of 1948 for the nominal GNP-growth equation, and the second quarter of 1948 for the rate of change of the unemployment-rate equation (equation [3]). The first observation for the rate of change of the GNP-deflator equation was the first quarter of 1952. Selection of these periods was based upon the availability of reliable and representative data.[9] In all cases, the data used were quarterly observations and, wherever possible, they were on a seasonally unadjusted basis.[10] The estimation technique employed was ordinary least squares.

In estimating the equations, quarter-to-quarter changes in the natural logarithm of the published series were used

[8] All "growth rates" discussed in this paper are relative rates of change continuously compounded (i.e., first differences of natural logarithms).

[9] Rates of inflation prior to 1952 display far greater variation than those in later years. If they had been included in the calculations, they would have swamped the relationship. Results for the complete time period from 1947:IV through 1969:IV are, however, reported below for the sake of completeness.

[10] The most important exception is the GNP price deflator which exists solely on a seasonally adjusted basis. This need cause little concern, since price series in general display little seasonal variation.

for most of the data; namely, GNP, the GNP price deflator, both money supply series, the unemployment rate, the S&P Index, government receipts, and both series of government expenditures. The market yield on Treasury bills was converted into a quarterly rate continuously compounded. The other variables were left untransformed.

Two prior constraints were placed on the formal model presented here. The first consisted of the preselection of variables to be considered for use in the model. The second was the selection of the functional form in which these variables were "tested."

Preselection of variables was based upon their a priori relevance in simple Keynesian, Quantity Theory, or Efficient Markets models. Naturally, certain institutional variables were also considered. In order to represent a Keynesian-type model, we considered both the current and lagged values of government purchases of goods and services, government receipts, and budget outlays. For a Quantity Theory position, we considered current and lagged values of two money supply variables. In order to help separate real from nominal effects of changes in "aggregate demand," we included the unemployment rate in the inflation equation.

Stock market prices, according to Efficient Markets Theory, in part, reflect future real income. Interest rates, in the same vein, reflect anticipated changes in the price level. Thus, both of these variables were considered. For institutional reasons, we also considered a variable representing the number of man-hours lost due to strikes, quarterly dummies, and certain lagged values of dependent variables.

The functional forms chosen for the equations reflected conventional economic reasoning so far as this was consistent with reasonable simplicity.

Once the series and the forms in which they were to be entered were selected, our methodology was to permit each independent variable to "explain" as much of the variation of the dependent variable as it could in competition with the other variables. Those variables which contributed little to the explanatory power of the equation, and which, in addition, were statistically insignificant, were omitted from the final equations.

B. THE BASIC RESULTS

The results of the tests are as follows:[11]

$$\begin{aligned}\Delta LY = {} & \underset{(4.9)}{0.032} - \underset{(12.1)}{0.098}\, D_1 + \underset{(2.6)}{0.025}\, D_2 \\ & - \underset{(4.0)}{0.029}\, D_3 + \underset{(5.5)}{1.10}\, \Delta LM1 \\ & + \underset{(6.9)}{0.136}\, \Delta LG - \underset{(3.3)}{0.068}\, \Delta LG_{-1} \\ & - \underset{(1.9)}{0.039}\, \Delta LG_{-2} - \underset{(1.2)}{0.024}\, \Delta LG_{-3} \\ & - \underset{(3.7)}{0.045}\, \Delta SH + \underset{(2.2)}{0.068}\, \Delta LS\&P_{-1}\,;\end{aligned} \qquad [1]$$

interval: 1948:I–1969:IV; $\bar{R}^2 = .958$; $F = 198$; D-W $= 2.15$; S.E. of E $= 0.0131$.

$$\begin{aligned}\Delta LP = {} & \underset{(0.1)}{0.000059} + \underset{(2.7)}{0.30}\, \Delta LP_{-1} \\ & + \underset{(2.0)}{0.22}\, \Delta LP_{-2} + \underset{(1.7)}{0.038}\, \Delta LM1 \\ & + \underset{(2.7)}{0.31}\, i_{-1}\,;\end{aligned} \qquad [2]$$

interval: 1952:I–1969:IV; $\bar{R}^2 = .443$; $F = 15$; D-W $= 1.79$; S.E. of E $= 0.00272$.

$$\begin{aligned}\Delta LUR = {} & \underset{(1.1)}{0.057} + \underset{(2.0)}{0.18}\, D_1 - \underset{(2.9)}{0.20}\, D_2 \\ & + \underset{(1.0)}{0.091}\, D_3 - \underset{(6.5)}{3.0}\, \Delta Ly - \underset{(5.2)}{2.4}\, \Delta Ly_{-1} \\ & - \underset{(1.3)}{0.59}\, \Delta Ly_{-2} - \underset{(3.7)}{1.8}\, \Delta Ly_{-3}\,;\end{aligned} \qquad [3]$$

[11] Computations were performed via time sharing on a Burroughs 550 system operated by Data Re-

interval: 1948:II–1969:IV; $\bar{R}^2 = .799$; $F = 50$; D-W = 1.74; S.E. of E = 0.0790.

$$\begin{aligned}\Delta Ly = {} & \underset{(5.6)}{0.038} - \underset{(12.0)}{0.10\, D_1} + \underset{(1.7)}{0.017\, D_2} \\ & - \underset{(4.4)}{0.034\, D_3} + \underset{(4.0)}{0.91\, \Delta LM1} \\ & + \underset{(5.6)}{0.14\, \Delta LG} - \underset{(2.8)}{0.067\, \Delta LG_{-1}} \\ & - \underset{(0.9)}{0.020\, \Delta LG_{-2}} - \underset{(0.6)}{0.012\, \Delta LG_{-3}} \\ & - \underset{(3.7)}{0.046\, \Delta SH} + \underset{(1.8)}{0.063\, \Delta LS\&P_{-1}} \\ & - \underset{(1.4)}{0.45\, \Delta LP_{-1}} - \underset{(0.7)}{0.22\, \Delta LP_{-2}} \\ & - \underset{(0.8)}{0.39\, i_{-1}}\,; \end{aligned} \quad [4]$$

interval: 1948:I–1969:IV; $\bar{R}^2 = .956$; $F = 146$; D-W = 2.16; S.E. of E = 0.0133.

In [2], over the longer period 1947:IV–1969:IV, the results are

$$\begin{aligned}\Delta LP = {} & \underset{(1.0)}{0.0012} + \underset{(5.0)}{0.52\, \Delta LP_{-1}} \\ & + \underset{(0.7)}{0.076\, \Delta LP_{-2}} + \underset{(2.4)}{0.085\, \Delta LM1} \\ & + \underset{(0.7)}{0.10\, i_{-1}}\,; \end{aligned}$$

$\bar{R}^2 = .355$; $F = 13$; D-W = 2.00; S.E. of E = 0.00508.

Key to Variables

ΔLY = Quarterly change in the log of nominal GNP.

D_1 = Seasonal variable for the first quarter.

D_2 = Seasonal variable for the second quarter.

D_3 = Seasonal variable for the third quarter.

$\Delta LM1$ = Quarterly change in the log of the conventional money supply (currency plus demand deposits)* (revised).

$\Delta LM2$ = Quarterly change in the log of the conventional money supply plus time deposits adjusted* (revised).

ΔLG = Quarterly change in the log of federal government purchases of goods and services.

ΔLR = Quarterly change in the log of federal government receipts.

ΔLE = Quarterly change in the log of total federal government expenditures.

SH = A measure of the proportion of industrial man-hours lost due to strikes (man-days idle as a result of strikes and lockouts, divided by total manufacturing employment).†

ΔSH = Quarterly change in SH.

$\Delta LS\&P$ = Quarterly change in the log of Standard and Poor's Composite Index of Common Stock Prices (the "S&P 500").*

ΔLP = Quarterly change in the log of the GNP price deflator.

i = Market yield on 13-week Treasury bills (percent per annum),* converted to continuous compounding at a quarterly rate by means of the formula $-\log(1 - .91I/360)$, where I is the published figure.

UR = Unemployment rate (the number of unemployed divided by the total civilian labor force).†

ΔLUR = Quarterly change in the log of UR.

ΔLy = Quarterly change in the log of real GNP.

$\Delta LDJF_{-5/3}$ = Quarterly change in the log of the Dow Jones index of farm commodity futures prices (five months' maturity), based on middle-of-month daily data, lagged five months.

S.E. of E = Standard error of estimate.

Asterisks denote that quarterly data are means of seasonally unadjusted monthly figures; daggers that the numerator and denominator of a ratio are quarterly means of seasonally unadjusted monthly figures.

Figures in parentheses below the coefficient estimates are t-statistics.

sources, Inc., Lexington, Mass. On this machine, five significant digits of accuracy are stated to be guaranteed under all circumstances.

In each of the equations, it is extremely difficult to distinguish the pattern of the residuals from that which one would expect from a normal distribution. This is especially true for the nominal GNP-growth equation and for the real GNP-growth equation. For the inflation and unemployment-growth equations, a slightly greater preponderance of residuals is found in the central region of the distribution.

Earlier studies of price movements (stock market as well as goods prices), have found them to follow a stable Paretian distribution with a characteristic exponent in the upper range between one and two.[12] Distributions of the stable Paretian type other than the normal distribution tend to produce more residuals in the central region than the normal distribution.

It is also interesting to note that the GNP and unemployment equations described above are virtually as efficient as any derivative equations using "generalized least squares." The optimal autoregressive parameters, ρ, were estimated to be 0.979 and 0.995, respectively (both insignificantly different from one). For the inflation equation, the generalized least-squares procedure did not prove to be very successful. However, we found that deviations of ρ below unity did not materially affect the precision of the equation.

Table 1 lists the percentage of "explained" variance attributable to each variable for the three primary equations.

As stated earlier, a number of variables other than those included in the final equations were tested. In tables 2, 3, and 4 are lists of all those variables considered, their respective coefficients, and the t-statistics of those coefficients. The exclusion of each variable (or group of variables) from a final equation was based either on the t-statistic(s) or on the actual versus the a priori sign of the coefficient(s). In several cases, two highly collinear variables were tested simultaneously, and the one with the higher t-statistic was selected for inclusion in a final equation.

The final equations were also tested for temporal stability. In each case, the estimation period was divided into two subperiods of equal length, and the equation was tested to determine whether there had been a significant structural shift from one subperiod to the other. In the first two cases, there was no evidence that a significant shift had, in fact, occurred.

In the case of the nominal GNP-growth equation, only two slope coef-

TABLE 1

DECOMPOSITION OF VARIANCE

GNP Growth [1]		Inflation [2]		Change in Unemployment Rate [3]	
Variable	%	Variable	%	Variable	%
D_1	33	ΔLP_{-1}	33	D_1	5
D_2	4	ΔLP_{-2}	18	D_2	10
D_3	9	$\Delta LM1$	15	D_3	1
$\Delta LM1$	14	i_{-1}	34	ΔLy	37
ΔLG	19	....	.....	ΔLy_{-1}	27
ΔLG_{-1}	6	....	.....	ΔLy_{-2}	2
ΔLG_{-2}	2	....	.....	ΔLy_{-3}	16
ΔLG_{-3}	1	....	.....	....	.....
ΔSH	8	....	.....	....	.....
$\Delta LS\&P_{-1}$	3	....	.....	....	.....
Total	100	....	100	....	100

NOTE.—Each percentage represents the square of the partial correlation coefficient of each variable as a ratio to the sum of squares of all partial correlation coefficients.

[12] A characteristic exponent of one describes the Cauchy distribution, and an exponent of two describes the normal distribution. For some empirical evidence on the statistical structure of price changes, see Fama.

TABLE 2

EQUATION [1]

$$\Delta LY = \underset{(4.9)}{0.032} - \underset{(12.1)}{0.098}\, D_1 + \underset{(2.6)}{0.025}\, D_1 - \underset{(4.0)}{0.029}\, D_3 + \underset{(5.5)}{1.10}\, \Delta LM1 + \underset{(6.9)}{0.14}\, \Delta LG - \underset{(3.3)}{0.068}\, \Delta LG_{-1}$$

$$- \underset{(1.9)}{0.039}\, \Delta LG_{-2} - \underset{(1.2)}{0.024}\, \Delta LG_{-3} - \underset{(3.7)}{0.045}\, \Delta SH + \underset{(2.2)}{0.068}\, \Delta LS\&P_{-1}$$

INTERVAL: 1948: I–1969: IV; S.E. of E = 0.0131

Variable	$\Delta LM1$	$\Delta LM1_{-1}$	$\Delta LM1_{-2}$	$\Delta LM1_{-3}$	$\Delta LM1_{-4}$	$\Delta LM2$	$\Delta LM2_{-1}$	$\Delta LM2_{-2}$	$\Delta LM2_{-3}$	$\Delta LM2$
In place of	$\Delta LM1$					$\Delta LM1$				$\Delta LM1$
Coefficient estimate	1.05	−0.39	0.46	−0.31	0.29	0.63	−0.58	0.59	−0.056	0.45
	(4.4)	(1.6)	(1.9)	(1.3)	(1.2)	(2.7)	(2.0)	(1.9)	(0.2)	(2.6)
S.E. of E	0.0127					0.0145				0.0148
Interval	1948: II–1969: IV					Same as [1]				Same as [1]

Variable	ΔLY_{-1}	ΔLY_{-2}	ΔLY_{-3}	SH	SH_{-1}	ΔLE	ΔLE_{-1}	ΔLE_{-2}	ΔLE_{-3}
In place of	(In addition)			ΔSH		ΔLG, ΔLG_{-1}, ΔLG_{-2}, and ΔLG_{-3}			
Coefficient estimate	−0.046	−0.024	−0.14	−0.034	0.057	0.056	−0.098	−0.0064	−0.029
	(0.6)	(0.3)	(1.7)	(2.4)	(4.0)	(1.9)	(3.3)	(0.2)	(0.9)
S.E. of E	0.0130			0.0130		0.0159			
Interval	Same as [1]			Same as [1]		Same as [1]			

Variable	ΔLR	ΔLR_{-1}	ΔLR_{-2}	ΔLR_{-3}	$\Delta LS\&P$	$\Delta LS\&P_{-1}$	$\Delta LS\&P_{-2}$	$\Delta LS\&P_{-3}$	LUR_{-1}
In place of	ΔLG, ΔLG_{-1}, ΔLG_{-2}, and ΔLG_{-3}				$\Delta LS\&P_{-2}$				(In addition)
Coefficient estimate	0.086	0.016	0.049	0.059	−0.033	0.075	0.0044	0.0024	0.0077
	(4.3)	(0.9)	(2.6)	(3.3)	(1.0)	(2.1)	(0.1)	(0.1)	(1.2)
S.E. of E	0.0149				0.0132				0.0130
Interval	Same as [1]				Same as [1]				Same as [1]

TABLE 3

EQUATION [2]

$$\Delta LP = \underset{(0.1)}{0.000059} + \underset{(2.7)}{0.30\,\Delta LP_{-1}} + \underset{(2.0)}{0.22\,\Delta LP_{-2}} + \underset{(1.7)}{0.038\,\Delta LM1} + \underset{(2.7)}{0.31 i_{-1}}$$

INTERVAL: 1952: I–1969: IV; S.E. of E = 0.00272

Variable...	$\Delta LM1$	$\Delta LM1_{-1}$	$\Delta LM1_{-2}$	$\Delta LM1_{-3}$	ΔLP_{-1}	ΔLP_{-2}	ΔLP_{-3}
In place of.	$\Delta LM1$				ΔLP_{-1} and ΔLP_{-2}		
Coefficient estimate.	0.027	0.031	−0.015	0.014	0.40	0.15	0.056
	(0.9)	(1.0)	(0.5)	(0.5)	(3.4)	(1.4)	(0.5)
S.E. of E..	0.00275				0.00263		
Interval...	Same as [2]				1952: II–1969: IV		

Variable...	Constant	D_1	D_2	D_3	ΔSH	ΔLY	$\Delta LM2$
In place of.	Constant				(In addition)	(In addition)	$\Delta LM1$
Coefficient estimate.	−0.00086	0.0017	0.00069	0.00081	−0.00016	−0.0080	0.013
	(0.6)	(1.1)	(0.4)	(0.6)	(0.1)	(1.4)	(0.4)
S.E. of E..	0.00273				0.00274	0.00270	0.00278
Interval...	Same as [2]				Same as [2]	Same as [2]	Same as [2]

Variable...	$\Delta LM1$	UR_{-1}	UR_{-2}	UR_{-3}	UR_{-4}	$\Delta LDJF_{-5/3}$
In place of.	$\Delta LM1$ and i_{-1}		(In addition)			i_{-1}
Coefficient estimate.	0.048	−0.041	−0.039	0.016	0.021	−0.0059
	(2.1)	(0.8)	(0.7)	(0.2)	(0.3)	(0.5)
S.E. of E..	0.00285	0.00271				0.00286
Interval...	Same as [2]	Same as [2]				Same as [2]

TABLE 4

EQUATION [3]

$$\Delta LUR = \underset{(1.1)}{0.057} + \underset{(2.0)}{0.18\,D_1} - \underset{(2.9)}{0.20\,D_2} + \underset{(1.0)}{0.091\,D_3}$$

$$- \underset{(6.5)}{3.0\,\Delta Ly} - \underset{(5.2)}{2.4\,\Delta Ly_{-1}} - \underset{(1.3)}{0.59\,\Delta Ly_{-2}} - \underset{(3.7)}{1.8\,\Delta Ly_{-3}}$$

INTERVAL: 1948: II–1969: IV; S.E. of E = 0.0790

Variable.......	ΔSH	$\Delta LM1$	ΔLG	ΔLG_{-1}	ΔLG_{-2}	ΔLy	ΔLy_{-1}
In place of.....	(In addition)	(In addition)	(In addition)			ΔLy, ΔLy_{-1}, ΔLy_{-2}, ΔLy_{-3}	
Coefficient estimate........	−0.097	−0.21	0.076	0.16	−0.086	−2.6	−2.4
	(1.3)	(0.2)	(0.6)	(1.1)	(0.6)	(5.4)	(4.9)
S.E. of E......	0.0787	0.0795	0.0794			0.0850	
Interval.......	Same as [3]	Same as [3]	Same as [3]			Same as [3]	

Variable.......	ΔLP	ΔLP_{-1}	ΔLP_{-2}	ΔLy_{-1}	ΔLy_{-2}	$\Delta Ly\Delta_{-3}$	ΔLy_{-4}
In place of.....	(In addition)			ΔLy, ΔLy_{-1}, ΔLy_{-2}, ΔLy_{-3}			
Coefficient estimate........	−0.87	−2.5	0.56	−2.1	−0.69	−0.95	0.29
	(0.5)	(1.2)	(0.3)	(3.5)	(1.2)	(1.6)	(0.5)
S.E. of E......	0.0788			0.0977			
Interval.......	Same as [3]			Same as [3]			

ficients could be interpreted as being somewhat different between the two subperiods. The third-quarter dummy variable appears to be more negative in the latter half of the period and the coefficient of the lagged stock index variable also may have decreased in size. These observations are not sufficiently important to justify an alteration of the full-period estimate.

With respect to the rate of change of GNP-deflator equation, there was no evidence of any intertemporal shifts within the period of estimation. The unemployment-rate equation does, on the other hand, appear subject to substantial instability in the individual coefficients. The seasonal pattern of the dummies is radically altered, as is the pattern of the growth rates of real GNP. However, the sum of each group of these coefficients does not appear to have changed. It is quite conceivable that the highly multicollinear variables have taken on different portions of the variance with little overall effect.

C. POLICY REQUIREMENTS

To the extent that the OMB model can be used for policy purposes, certain conditions should be met. First, if monetary policy is to be effective in controlling nominal GNP growth, then one has to presume that the policy makers do, in fact, directly influence the growth of the quantity of money and have influenced this growth in the past to the same extent. Likewise, if fiscal policy is to be effective, a similar presumption must be made. There is, of course, a great deal of evidence to suggest that policies do, in fact, influence the growth of both government purchases and the money supply.

No change in the methodology of this study would be warranted even if the money supply and government purchases were completely endogenous, for example, both passively responding to changes in GNP. The statistical relationships found would still be valid. On the other hand, the extent to which these variables are endogenous does, of course, reduce the usefulness of these results to policy makers.

Second, policy use of these statistical relationships must also assume that the relationships among the variables are basically the same now as in the past.

Both of these requirements are necessary conditions for this model to be an effective policy tool.[13] Each of these requirements, of course, applies equally to other formal models. For many other formal models, a great many more conditions are required.

It is also important to recognize that a model becomes less and less reliable the further the explanatory variables deviate from their historical means. Thus, inferences from the OMB model as to the outcome with, say, an 8 percent growth in the money supply, or other extreme observations, may be highly unreliable.

D. INTERPRETATION OF FINDINGS

The following inferences can be drawn from the above results:

1. Fiscal policy, as represented by federal government purchases of goods and services, provides a temporary stimulus to the level of GNP. After three quarters, the cumulative effect is insignificantly different from zero.[14] Interpreting the re-

[13] They are not sufficient conditions. Statistical misspecifications and measurement errors in the data may still result in estimates that are not efficient enough to make the model reliable as a policy aid.

[14] This finding, in a slightly different context, was reported earlier by Andersen and Carlson, the originators of the extremely important model popularized by the Federal Reserve Bank of Saint Louis (see Leonall Andersen and Keith Carlson, "A Monetarist

sults differently, a \$1.00 increase in government purchases is associated with approximately a \$1.00 increase in current GNP and a reduction in GNP of \$1.00 in the future three quarters. We assume for the purposes of this discussion that the government can and has controlled its purchases.[15]

2. Monetary policy, as represented by changes in the conventionally defined money supply, has an immediate and permanent impact on the level of GNP. For every dollar increase in the money supply, GNP will rise by about \$4.00 or \$5.00 and not fall back in the future. Alternatively, every 1 percent change in the money supply is associated with about a 1 percent change in GNP. Again we assume, for expositional purposes, that the government has controlled and does control the money supply.

3. Movements in the S&P Index provide reliable information with respect to future changes in GNP. The average lag is approximately three months.

4. The Treasury bill rate provides reliable information with respect to the future rate of inflation.

5. The conventional money supply data are empirically superior to data for the conventional money supply plus time deposits. Government purchases are also empirically superior to either budget outlays or receipts.

6. The evidence displayed here does not support a significant partial relationship between the rate of change of the GNP price deflator and the rate of unemployment. The results do not confirm the existence of a "Phillips Curve."

7. Labor-hoarding hypotheses are provided with empirical support by the finding that changes in the rate of unemployment are closely related to lagged changes in real GNP.

E. THE "LAG" IN EFFECT OF MONETARY POLICY

Perhaps the most striking result of the tests is the absence of a significant relationship between lagged growth rates of the money supply and the growth rate of nominal GNP. Even though several other writers have been unable to find a lag, the presumption of lags is still perhaps the most well-entrenched tenet of the modern Quantity Theory school.[16] In fact, there are some observers who contend that a failure to find the lagged relationship would be important evidence against the Quantity Theory premise that changes in the money supply are a cause of changes in nominal GNP.

Actually, there is no necessary linkage between temporal precedence and causality. Examples to the contrary are available. Given that high interest rates precede high rates of inflation, it is still eminently reasonable that rates of inflation are the direct "cause" of high interest rates. In a similar vein, stock price movements can precede changes in profits, and yet still be "caused" by these profits. More generally, Tobin and Brainard[17] describe "reasonable" struc-

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Model for Economic Stabilization," *Federal Reserve Bank of Saint Louis Review* 52 [April 1970]: 7–25).

[15] In the Saint Louis model, the variables appear in the form of first differences, and not first differences in the logs. For the fiscal variables (but not the monetary variable), the Saint Louis functional form appears preferable. Fortunately, the differences in results are insignificant.

[16] Other writers who have failed to find evidence of a significant lag include Ernest Tanner, "Lags in the Effects of Monetary Policy: A Statistical Investigation," *American Economic Review* 59 (December 1969): 794–805; Julius Shiskin, "Economic Policy Indicators and Cyclical Turning Points," *Business Economics* 5, no. 4 (September 1970): 99–102; and Geoffrey Moore in unpublished papers.

[17] William Brainard and James Tobin, "Pitfalls in Financial Model Building," *American Economic Review* 58 (May 1968): 99–122, esp. pp. 120–22. James

tural models with built-in causation. They then show how these models can provide a complete menu of lead and lag relationships in the actual data.

In order to help dispel some of the doubts associated with our findings of no lagged money relationships, several additional experiments were performed.

First, the annual growth rate of GNP was related to eight consecutive quarterly growth rates of the money supply

$$\sum_{t=\tau-3}^{t=\tau} \Delta LY_t = \text{intercept} + \sum_{j=0}^{j=7} a_j \Delta LM_{\tau-j} + \text{residual}\,,$$

where ΔLY_t represents the change in the log of nominal GNP from quarter $t-1$ to quarter t, and (*a*) $\tau = 11, 15, 19, \ldots$; (*b*) $\tau = 10, 14, 18, \ldots$; (*c*) $\tau = 9, 13, 17, \ldots$; (*d*) $\tau = 8, 12, 16, \ldots$; interval: 1949–1969; $\tau = 1$ corresponds to 1947:II.

To help interpret this equation, the change in log GNP from 1963:III to 1964:III, for example, was estimated as a function of the following quarterly relative changes in log money: 1964:II–III, 1964:I–II, 1963:IV–1964:I, 1963:III–IV, 1963:II–III, 1963:I–II, 1962:IV–1963:I, and 1962:III–1962:IV. In other words, the first four of the quarterly money supply variables overlapped exactly with the annual GNP variable; the second four overlapped exactly with the corresponding GNP variable for the preceding year.

This test was carried out four different ways, representing the four alternative annual quarter-to-quarter changes possible for GNP (table 5). All in all, these results do not demonstrate the existence of a lag.

In another experiment, we partitioned the difference between the Saint Louis "lag" results and our results. The basic differences between the Saint Louis procedures and ours are (*a*) the use of the Almon lag procedure; (*b*) the use of seasonally adjusted data; and (*c*) the functional form, etc. The various lag structures obtained are shown in table 6.

As is readily apparent from the table, the concurrent money supply variable picks up additional strength as the Al-

TABLE 5

TEST FOR LAG IN RELATIONSHIP BETWEEN GROWTH RATES OF NOMINAL GNP AND MONEY SUPPLY

Annual Change in Log GNP, Quarter to Quarter	Average of the Coefficients of the 4 Concurrent Variables	Average of 4 Lagged Coefficients
a) 4–4	1.69	−0.80
b) 3–3	0.67	0.17
c) 2–2	0.84	0.04
d) 1–1	0.84	0.18
Average	1.01	−0.10

TABLE 6

ESTIMATED LAG DISTRIBUTION OF TOTAL EFFECTS OF MONEY ON GNP

EQUATION	MONEY COEFFICIENT BY QUARTER FRACTION OF TOTAL				
	t	$t-1$	$t-2$	$t-3$	$t-4$
Original Saint Louis	0.24	0.34	0.28	0.14	0.00
Original Saint Louis, ex Almon	0.40	0.07	0.43	0.16	−0.06
Original Saint Louis ex seasonally adjusted and ex Almon	0.83	−0.72	1.05	−0.45	0.29
OMB results	0.96	−0.36	0.42	−0.28	0.26

Tobin, "Money and Income: *Post Hoc Ergo Propter Hoc*," *Quarterly Journal of Economics* 84 (May 1970): 301–17.

mon lag procedure is dropped, as we move to seasonally unadjusted data, and, finally, as we go to the OMB functional form. The coefficient of the concurrent variable is very close to unity by the time the Almon procedure and seasonal adjustments have been removed.

With the full Saint Louis technique, the first lag carries the largest coefficient, but it declines when the Almon procedure and seasonal adjustments are removed. Except when seasonally adjusted data or Almon lags are used, the lagged values of changes in the money supply almost exactly offset each other.

It is conceivable that someone might feel that the stock market variable is acting in some sense as a proxy for lagged money supply changes. This view is not supported by the empirical evidence. Even when the stock market variable is omitted, the sum of the coefficients of the lagged money supply variables remains slightly negative.

F. EXOGENEITY OR ENDOGENEITY OF MONEY

Statistical correlations do not demonstrate causal flows any more than do lead-and-lag relationships. From the results presented earlier, it could just as well be that money supply changes result from changes in GNP as the reverse. Although, in general, we are unable to answer this question of cause and effect, we can respond to three of the more common routes suggested.

It may be argued that the money multiplier acts as an offset to variations in the effective reserve base/GNP ratio. Although the Federal Reserve may control the effective reserve base, it is often argued that it cannot control the money supply because the money multiplier is free to move.

On a priori grounds, this particular view of endogeneity is somewhat suspect because part of the money multiplier is clearly affected by policies of the Federal Reserve (the discount mechanism, Regulation Q, and so on).[18] On empirical grounds, it is even more suspect.

In the first place, percentage changes in effective reserves are not very highly correlated with percentage changes in the money multiplier. An even more convincing exercise arises when we substitute the effective reserve base for the money supply in the GNP equation:

$$
\begin{aligned}
\Delta LY = {} & \underset{(8.7)}{0.049} - \underset{(12.3)}{0.11}\, D_1 - \underset{(0.9)}{0.006}\, D_2 \\
& - \underset{(7.4)}{0.049}\, D_3 + \underset{(3.3)}{0.76}\, \Delta LMB \\
& + \underset{(6.2)}{0.135}\, \Delta LG - \underset{(3.5)}{0.081}\, \Delta LG_{-1} \\
& - \underset{(1.5)}{0.034}\, \Delta LG_{-2} - \underset{(0.7)}{0.016}\, \Delta LG_{-3} \\
& - \underset{(3.0)}{0.041}\, \Delta SH + \underset{(3.5)}{0.11}\, \Delta LS\&P_{-1} ,
\end{aligned}
$$

same interval as equation (1); $\bar{R}^2 = .949$; $F = 161$; D-W = 2.34; S.E. of E = 0.0144, where ΔLMB = change in the log of effective reserves (based on quarterly means of seasonally unadjusted monthly data). These data were furnished us by the staff of the Federal Reserve Bank of Saint Louis.

As is apparent from the above equation, effective reserves have a powerful effect even when we do not hold the money multiplier constant. From this, one could infer that, even allowing for offsets due to the money multiplier, there is still a statistically significant effect for changes in effective reserves.

The second route via which the money supply may be thought to be endogenous

[18] Work on this topic has been reported by investigators at the Federal Reserve Bank of Saint Louis and by Karl Brunner and Allan Meltzer.

is through coseasonal variation. The argument proceeds along the following lines: Because GNP has a distinct seasonal variation and the Federal Reserve has historically followed an "even-keel" interest rate policy, changes in the money supply represent a response of the Federal Reserve to GNP, and not the reverse.

Surprising as it may be, the seasonal variations of GNP and the money supply are not the same. In table 7 is a list of the seasonal deviations of each series and their *t*-statistics.

GNP and money move in the opposite direction two out of four quarters. The magnitudes of their movements, even when they do move in the same direction, are quite different. More evidence is clearly required to push the case that seasonal variations give rise to the correlation between GNP and money.

Perhaps somewhat more convincing evidence against this suggested seasonal route for endogeneity was found in the section of "lags." The relationship between annual changes in GNP and changes in the money supply was found to be almost identical with that found for the quarterly relationships. "Seasonal" fluctuations can hardly be a major cause of annual relationships.

TABLE 7

PERCENTAGE SEASONAL VARIATION RELATIVE TO THE MEAN: 1947–69

Growth Rate of:	Quarter			
	I	II	III	IV
Nominal GNP....	−9.66 (22.5)	3.40 (7.9)	−0.16 (0.4)	6.44 (15.0)
Money supply......	−0.64 (4.0)	−1.60 (10.0)	0.17 (1.1)	2.07 (12.9)

Incidentally, as far as seasonal variations and endogeneity are concerned, the case appears far stronger for government purchases of goods and services. GNP and federal government purchases of goods and services have almost the same seasonal pattern.

There is another problem of interpretation that arises from the possibility that the money supply variable and the government purchases variable might be causally related. In fact, the adjusted coefficient of determination, $\bar{R}^2$, between the concurrent magnitudes of the two policy variables is about 10 percent, and the slope coefficient of the purchases variable, 0.052, is significantly different from zero. However, when three lagged terms in government purchases are included, the sum of the slope coefficients is 0.01, which is not significantly different from zero.

Assuming that government authorities control fiscal variables, the evidence here implies that they cannot permanently affect the level of GNP without constantly shifting the share of GNP devoted to government purchases of goods and services.

Finally, it is again important to note that the results can only show statistical correlations and do not purport to show cause and effect. This section has attempted to make somewhat less plausible three of the more commonly hypothesized routes for endogeneity.

G. PRECISION

For each of the equations, an estimate of "goodness-of-fit" was obtained. On a quarterly basis, the standard error of estimate, the figure reported, gives some indication of the accuracy of the model over the estimation period. Additional tests were performed in order to give a sense of the accuracy of this model in relation to other models. As a further test of precision, we were also able to present a forecast generated by the model outside the sample period, namely, the four quarters of 1970.

Table 8 presents three columns of figures. The first column is the standard error of estimate for each of the quarterly equations reported above. The second column is the expected standard error of estimate of a year's data based upon the assumption that the errors are normally distributed. This column is calculated by multiplying the first column by the square root of the number of periods averaged, that is, by the square root of 4, or 2. The third column is the measured standard deviation of the annual average error based upon actual experience. All of the above measures of precision are computed for each of the four equations.

The annual standard errors of estimate in column 3 are slightly different from those which one would expect from the quarterly equations (col. 2).[19] It is possible that these discrepancies result from divergence from normality, or from some higher order serial dependence in the residuals.

One other important test of a model of this or any other type is to evaluate medium-term forecasts during the sample period and see whether the model behaves in a "reasonable" fashion. Starting with the actual fourth-quarter figures for the preceding year, annual estimates were made, assuming knowledge of all exogenous or predetermined variables. Forecasted values of dependent variables as yet unobserved were computed from the other equations. The forecasted values of the dependent variables were used. In table 9, we have listed the actual and forecasted values by year for nominal GNP, the GNP deflator, real GNP, and the unemployment rate. From this table the reader should have a clear picture of the accuracy of the model. In the GNP equation, any forecast error tends to be carried forward over time as a random walk. There is little evidence of drift. However, the GNP deflator, because of the autoregressive structure of the equation, is subject to errors which tend to grow with the span of the forecast.

Because our GNP-growth equation is predicated upon original data while other models use seasonally adjusted data, it is difficult to compare quarterly errors of estimate. In any case, the larger GNP models have few, if any, degrees of freedom left, so that it would hardly be appropriate to compare the precision of their estimates with that of ours. Were we able to duplicate their forecasts outside of the sample period, a comparison of our model with any model would be valid. However, we question whether it would be worth the expense to reestimate the larger models and generate forecasts outside of the same period.

The only study of the precision of large-scale econometric models with

TABLE 8

COMPARISON OF ESTIMATION ERRORS

Equation	Quarterly (1)	$(1)\times\sqrt{4}$ (2)	S.E. of Annual Estimates* (3)
1. Nominal GNP....	0.0131	0.0262	0.0219
2. Inflation.........	0.0027	0.0054	0.0059
3. Unemployment...	0.0790	0.1580	0.1263
4. Real GNP........	0.0133	0.0267	0.0202

* These figures are based upon fourth-quarter-over-fourth-quarter comparisons, and are adjusted for degrees of freedom.

[19] Standard errors of estimate for year-over-year comparisons would be somewhat smaller.

TABLE 9

EX POST FORECASTS OF OMB MODEL

Year	Nominal GNP (Billion $)		Inflation (at Annual Rate) (%)		Real GNP (Billion $)		Unemployment Rate (%)	
	Actual	Forecast	Actual	Forecast	Actual	Forecast	Actual	Forecast
1948	258	252	...	...	...	...	...	...
1949	257	258	...	...	...	...	...	...
1950	285	276	...	...	...	...	...	...
1951	328	328	...	...	...	...	...	...
1952	345	346	1.8	2.1	395	394	3.0	3.6
1953	365	367	1.1	1.8	413	411	2.9	2.7
1954	365	371	1.6	0.6	407	418	5.6	4.2
1955	398	395	2.0	1.0	438	437	4.4	4.5
1956	419	420	4.1	1.9	446	454	4.1	4.3
1957	441	436	3.2	2.6	452	449	4.3	4.7
1958	447	459	2.1	1.9	448	460	6.8	5.6
1959	484	482	1.5	2.0	476	474	5.5	5.8
1960	504	502	1.8	1.8	488	486	5.5	5.9
1961	520	529	1.1	1.7	497	503	6.7	6.6
1962	561	552	1.1	1.8	530	520	5.5	5.9
1963	591	604	1.4	2.0	551	561	5.7	5.0
1964	633	637	1.7	2.3	581	583	5.2	5.2
1965	685	684	1.7	2.5	618	614	4.5	4.6
1966	750	742	3.4	2.7	658	655	3.8	3.6
1967	794	807	3.4	3.3	675	685	3.8	3.7
1968	865	860	4.0	4.0	707	703	3.6	4.1
1969	931	921	4.9	4.3	727	722	3.5	3.6
1970*	977	979	5.1	5.0	724	727	4.9	4.4
Mean absolute error	$5.4 (1.0% of mean)		0.66% per annum (27% of mean)		$5.2 (1.0% of mean)		0.4 percentage points (8.2% of mean)	

* Quarter-by-quarter ex post forecasts for 1970 are detailed below.

which we are familiar is that by Charles Nelson.[20] Professor Nelson presented a great deal of evidence which suggests that the conditional forecasting properties of one of these large-scale models outside the sample period are scarcely better than extrapolations of purely stochastic linear processes devoid of economic meaning. For some variables (such as GNP, the GNP deflator, and the unemployment rate), the naïve models have smaller mean square errors. Indeed, for the unemployment rate, the naïve model has a smaller mean square error even within the sample period. Nelson's findings are more remarkable when we bear in mind that actual knowledge of exogenous data was employed in his comparisons. Extensive tests of our model against these large-scale models would be prohibitively expensive, both financially and in terms of time. We do not feel the effort can be justified after reading Nelson's paper.

Fortunately, the Saint Louis model is of a much lower order of complexity than the large-scale models. The precision of the nominal-GNP-growth equation can

[20] Charles Nelson, "The Prediction Performance of the FRB-MIT-Penn Model of the U.S. Economy," Report 7117, Center for Mathematical Studies in Business and Economics, University of Chicago (April 1971).

be compared, albeit imperfectly, with that of a pair of Saint Louis–type equations.

In order to make the comparison, we need to examine the annual standard errors of estimate and the annual standard deviations of both the seasonally adjusted and unadjusted data. It is perhaps not well known that the process of seasonal adjustment tends to reduce the standard deviation of annual percentage changes in GNP measured on a quarter-to-quarter basis. The difference averages about 1 percent over the postwar period. Evidently these so-called seasonal factors operate so as to smooth even annual data. Thus, even if the percentage of variance explained by each model were the same, the model which uses seasonally adjusted data would have a lower standard error of estimate.

The first comparison is with the Saint Louis model itself over the period of estimation selected by Saint Louis: 1953:I–1969:IV. Both the formal model presented here and the Saint Louis model include a current money supply variable and current and lagged values of government purchases of goods and services. The Saint Louis model includes several lagged money supply terms, whereas our model includes the lagged stock price index variable and an allowance for strikes.

In spite of the fact that the unadjusted data series is more volatile than the seasonally adjusted series, the standard error of annual GNP growth (fourth quarter to fourth quarter) for our model is about 10 percent smaller than for the Saint Louis model, 0.0147 as opposed to 0.0164. When calculated over the full time period, our model shows an even greater superiority over Saint Louis. However, when the same comparisons are made on dollar changes in GNP, the Saint Louis model is superior to ours.

The second comparison is between our model and what we shall refer to as the Saint Louis *optimum optimorum*. The latter was estimated over the full time period, using seasonally adjusted data. The variables were used in the form of changes in logs, and the equation included three lagged money supply terms, as well as all the variables found to be influential in the OMB model. The coefficients of the lagged monetary and fiscal variables were completely free to take on any value. In table 10 we have listed the annual standard errors of estimate for both the *optimum optimorum* model and for our own. These calculations have been carried out for each of the four alternative annual quarter-over-quarter changes. In spite of the significant handicaps, the OMB model is at least equal in precision to the *optimum optimorum* which we constructed.

A final point of interest is the short, but important, interval following our sample period. Due to the fact that the OMB model requires seasonally unad-

TABLE 10

ANNUAL STANDARD ERRORS OF ESTIMATE

Model	Quarter				Average
	IV over IV	III over III	II over II	I over I	
Optimum optimorum	0.0212	0.0217	0.0215	0.0191	0.0210
OMB equation 1....	0.0195	0.0211	0.0233	0.0192	0.0208

justed data for inputs, we restricted the length of our original estimation period to the fourth quarter of 1969. Since that time, we have been able to estimate seasonally unadjusted data for GNP and federal purchases of goods and services by quarter for the year 1970.[21] Table 11 contains a comparison of the estimated actual data with our conditional forecasts of nominal GNP, the rate of change of the GNP deflator, real GNP, and the unemployment rate. The forecasts are those which would have been made at the end of 1969, assuming the ex post behavior of the policy variables, strikes, and the stock market.

III. SUMMARY AND CONCLUSIONS

A. REVIEW OF FINDINGS

1. Fiscal policy, as represented by federal purchases of goods and services (assumed exogenous), provides a powerful temporary stimulus to GNP. Over a year, the cumulative effect is near zero.
2. Changes in the conventionally defined money supply (for expositional purposes assumed exogenous) have an immediate and permanent impact on the level of GNP. Additional search yields little evidence of lags.
3. Movements in the stock market provide reliable information with respect to future changes in GNP.
4. The market rate of interest provides reliable information with respect to the future rate of inflation.
5. The conventional money supply data are empirically superior to data for the conventional money supply plus time deposits.
6. Government purchases are empirically superior to either budget outlays or receipts.
7. The precision of the nominal GNP equations is quite satisfactory.
8. The results with respect to the GNP deflator do not confirm the existence of a "Phillips Curve."
9. Labor-hoarding hypotheses are supported by the finding that changes in the rate of unemployment are closely related to lagged changes in real output.

B. GOODNESS OF FIT

In each of the three figures below (figs. 1–3), we have constructed two plots. The first plot compares actual and the predicted values by year over the estimation period. The second is a scatter diagram of the actual and the predicted values. A

[21] Official figures were not available at the time of this exercise.

TABLE 11

EX POST FORECASTS FOR 1970

Period	Nominal GNP (Billion $)		Inflation (at Annual Rate) (%)		Real GNP (Billion $)		Unemployment Rate (%)	
	Actual	Forecast	Actual	Forecast	Actual	Forecast	Actual	Forecast
70: I	228.8	229.6	6.2	5.0	172.6	173.8	4.5	4.6
70: II	244.2	242.5	4.2	4.9	182.3	181.2	4.7	4.2
70: III	246.1	245.3	4.5	4.9	181.6	181.1	5.2	4.3
70: IV	257.5	261.2	5.5	5.0	187.4	190.4	5.4	4.4
70: Yr	976.6	978.6	5.13	4.96	723.9	726.6	4.94	4.38

NOTE.—Where nonpredetermined dependent variables appeared in these calculations, the forecasted and not the actual values were used (e.g., lagged inflation, real GNP growth).

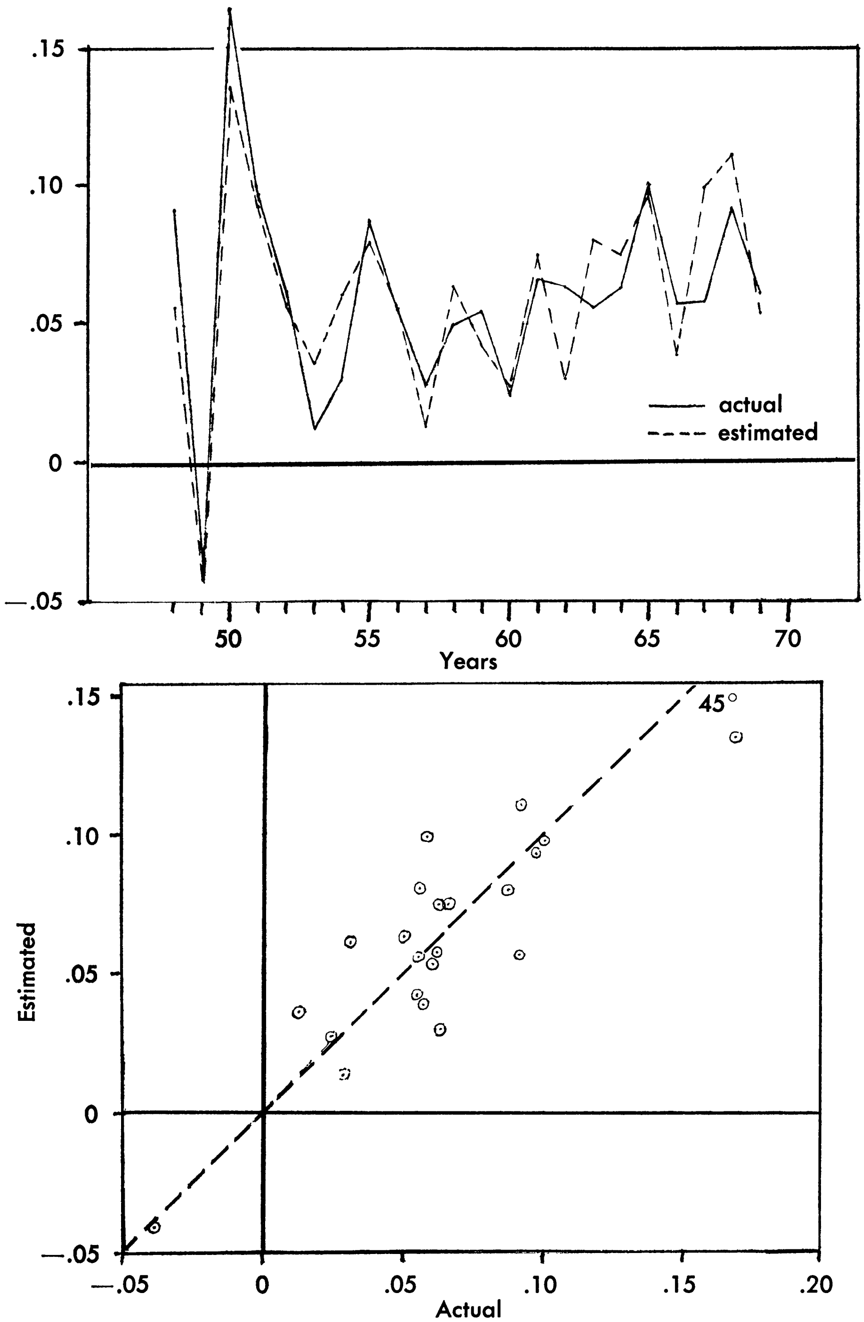

FIG. 1.—Growth rate of nominal GNP

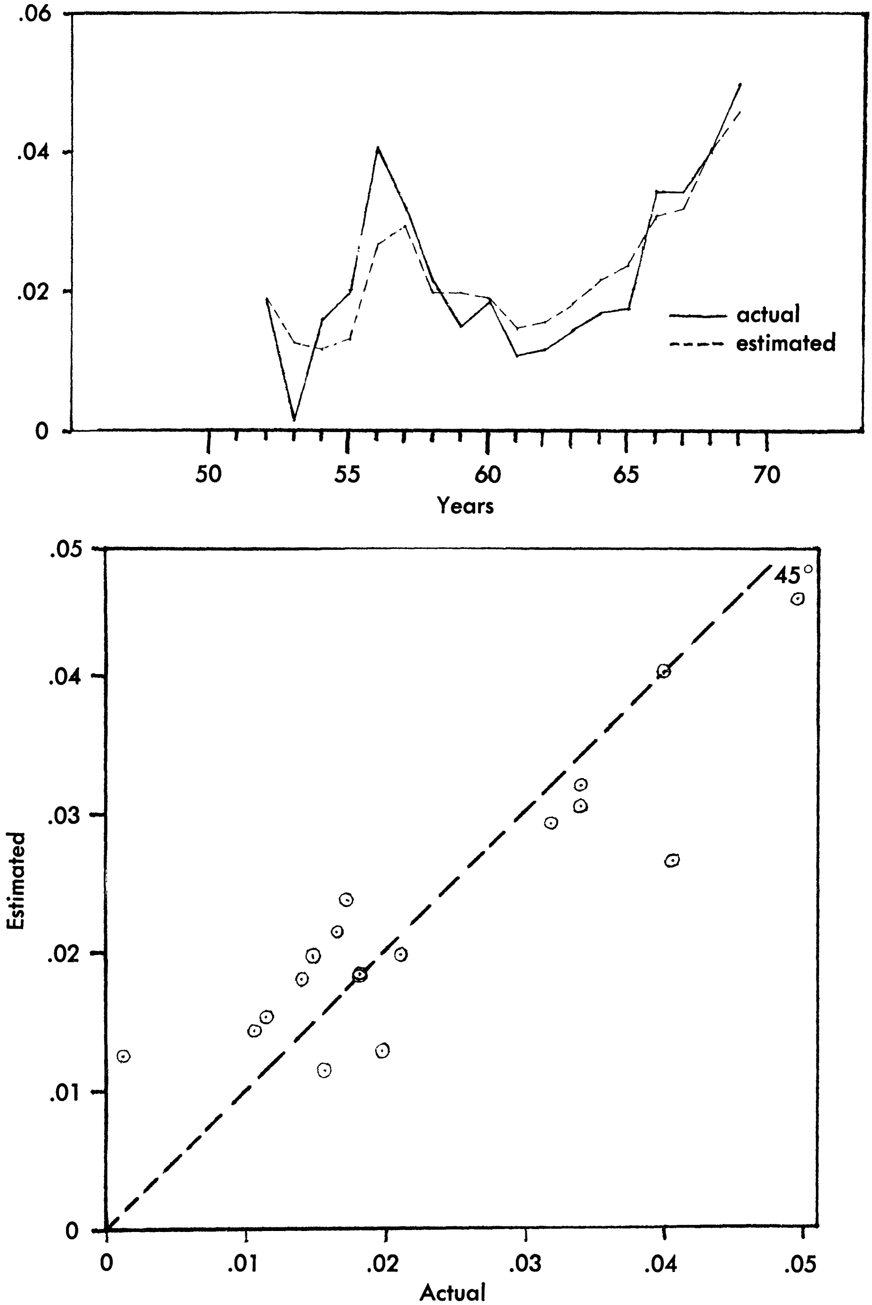

FIG. 2.—Rate of inflation

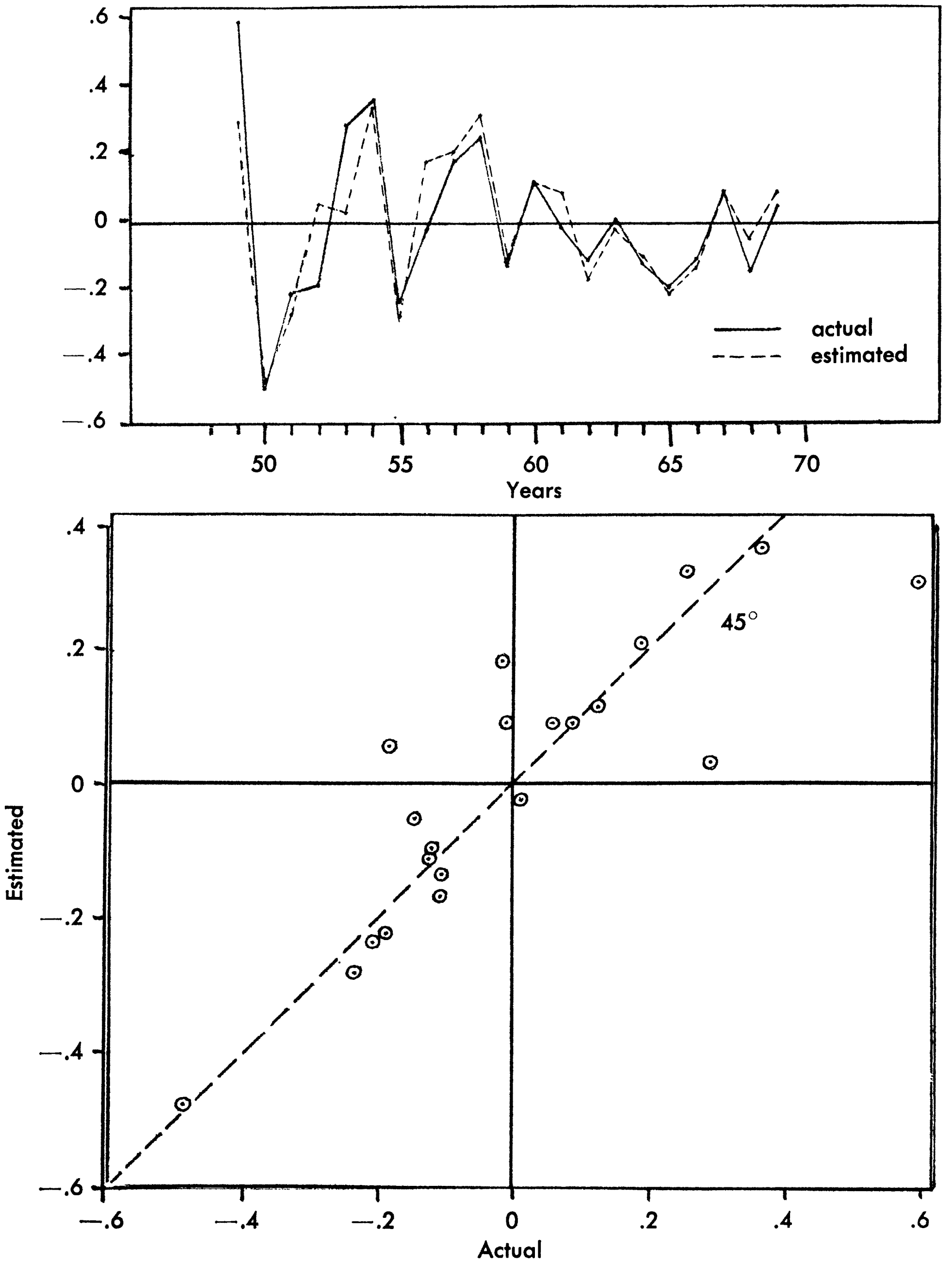

FIG. 3.—Rate of change of the unemployment rate

pair of plots is provided for each of three variables: the growth rate of nominal GNP, the rate of inflation, and the rate of change of the unemployment rate.[22]

The accuracy of the equations is readily visible from the plots. In addition, the lower set of plots shows quite clearly the absence of "underestimation bias," except possibly in the case of the rate of change of the GNP deflator. It is a well-known and well-documented fact that models frequently tend to underpredict large changes.[23] They gravitate toward the mean. This does not appear to be the case for this model.

[22] Growth rates are measured from fourth quarter to fourth quarter at annual rates.

[23] See Henri Theil, *Applied Economic Forecasting* (Chicago: Rand McNally & Co., 1966), p. 14 and chap. 2.

APPENDIX

THE DATA

TABLE A1

GROSS NATIONAL PRODUCT

Year	Quarter I	II	III	IV
1947	53.4	55.9	57.5	64.5
1948	59.4	62.1	65.3	70.7
1949	61.8	62.2	64.5	68.0
1950	64.0	66.8	73.5	80.5
1951	76.6	79.9	83.2	88.7
1952	82.0	83.3	85.7	94.4
1953	87.4	91.3	90.3	95.6
1954	86.5	89.7	90.0	98.6
1955	92.6	97.4	00.4	107.6
1956	98.6	102.9	104.1	113.7
1957	104.4	109.1	110.6	117.0
1958	103.9	108.8	111.7	123.0
1959	113.1	121.4	119.3	129.9
1960	120.5	125.6	124.6	133.1
1961	120.6	128.2	129.1	142.2
1962	131.3	139.6	138.1	151.5
1963	137.8	146.1	146.5	160.2
1964	148.5	157.1	156.3	170.6
1965	158.2	169.1	168.9	188.7
1966	176.2	187.4	186.3	199.8
1967	186.5	197.2	198.4	211.7
1968	199.9	217.3	215.8	232.0
1969	217.5	232.4	234.8	246.5

NOTE.—At quarterly rates, unadjusted for seasonal variation, in billions of current dollars.

TABLE A2

MONEY SUPPLY (DEMAND DEPOSITS PLUS CURRENCY)

Year	Quarter I	II	III	IV
1947	110.4	109.9	112.2	114.7
1948	113.7	110.5	111.8	113.3
1949	111.9	109.8	110.5	112.5
1950	112.7	112.1	114.3	117.5
1951	118.1	116.7	118.6	123.5
1952	124.7	123.1	124.5	128.7
1953	128.8	127.0	127.3	130.3
1954	130.3	128.1	129.3	133.4
1955	134.7	133.0	133.5	136.5
1956	136.8	134.7	134.6	138.0
1957	137.9	135.7	135.8	137.6
1958	136.9	136.5	137.9	142.2
1959	143.0	142.2	143.0	144.8
1960	142.7	139.9	140.4	143.5
1961	143.1	142.3	142.9	147.5
1962	147.2	145.9	145.4	149.5
1963	150.0	149.3	150.4	155.5
1964	155.4	154.3	156.7	162.4
1965	162.3	160.9	162.6	169.5
1966	170.6	170.2	169.5	173.8
1967	173.8	174.3	178.1	184.8
1968	185.6	187.0	190.7	198.5
1969	200.1	200.1	200.8	206.1
1970	206.3	207.5	209.7	216.5

NOTE.—Means of daily data unadjusted for seasonal variation, in billions of dollars.

TABLE A3

FEDERAL GOVERNMENT PURCHASES OF GOODS AND SERVICES

YEAR	QUARTER I	II	III	IV
1947	2.9	3.3	3.1	3.3
1948	3.0	3.8	4.2	5.5
1949	4.9	4.7	5.1	5.5
1950	4.7	3.7	4.3	5.7
1951	6.7	8.4	10.7	11.9
1952	11.6	12.5	13.4	14.3
1953	13.9	14.5	13.5	15.2
1954	12.6	12.0	10.9	11.9
1955	10.6	10.4	11.1	11.9
1956	10.6	11.5	10.8	12.7
1957	12.1	12.1	12.1	13.2
1958	12.4	13.0	13.3	14.9
1959	13.1	13.4	13.1	14.1
1960	12.7	12.8	13.8	14.2
1961	13.5	14.0	14.1	15.8
1962	15.2	15.9	15.3	17.1
1963	15.7	15.4	16.0	17.1
1964	15.9	16.6	15.8	16.8
1965	15.4	16.2	16.2	19.0
1966	17.6	18.9	20.1	21.2
1967	21.5	22.5	22.3	24.4
1968	23.0	25.5	24.3	26.6
1969	24.5	24.9	26.0	26.0

NOTE.—At quarterly rates, unadjusted for seasonal variation, in billions of dollars.

TABLE A4

STANDARD & POOR'S COMPOSITE INDEX OF STOCK PRICES

YEAR	QUARTER I	II	III	IV
1947	15.4	14.6	15.4	15.3
1948	14.4	16.1	16.0	15.6
1949	15.0	14.5	15.2	16.2
1950	17.1	18.3	18.3	19.8
1951	21.6	21.8	22.8	23.2
1952	23.9	23.9	15.0	25.1
1953	26.0	24.5	24.0	24.4
1954	26.0	28.4	30.8	33.5
1955	36.3	38.4	43.2	44.1
1956	45.4	47.0	48.0	46.1
1957	44.3	46.5	46.1	40.6
1958	41.5	43.6	47.5	52.3
1959	55.5	57.5	58.7	57.8
1960	56.3	56.1	55.7	55.3
1961	62.0	66.0	66.8	70.3
1962	69.9	62.2	57.8	59.6
1963	65.6	69.7	71.0	73.3
1964	77.5	80.3	82.9	84.8
1965	86.6	87.4	86.9	91.8
1966	91.6	88.1	81.4	79.8
1967	87.1	91.7	94.4	94.5
1968	91.6	98.0	99.9	105.2
1969	100.9	101.7	94.5	94.3
1970	88.7	79.2	78.7	86.2

NOTE.—Means of daily data.

TABLE A5

MAN-DAYS IDLE DUE TO STRIKES AND LOCKOUTS

YEAR	QUARTER I	II	III	IV
1947	1.223	6.410	2.820	1.066
1948	2.801	4.570	2.437	1.561
1949	1.620	3.260	3.587	8.373
1950	5.063	3.060	2.973	1.851
1951	1.640	1.837	2.353	1.807
1952	1.440	9.463	6.300	2.471
1953	1.240	3.863	2.820	1.700
1954	1.132	1.873	3.317	1.205
1955	0.892	2.977	3.050	2.480
1956	2.147	2.153	5.697	1.037
1957	0.782	1.883	1.967	0.860
1958	0.746	1.630	2.240	3.353
1959	1.477	2.760	12.143	6.610
1960	1.313	2.330	1.830	0.897
1961	0.612	1.418	1.787	1.612
1962	0.899	2.223	1.850	1.220
1963	1.408	1.306	1.382	1.269
1964	0.918	1.823	1.777	3.127
1965	1.650	2.093	2.670	1.352
1966	1.143	2.563	2.750	2.003
1967	1.344	3.960	4.449	4.288
1968	3.485	6.235	3.914	2.705
1969	2.717	4.407	3.380	3.786
1970	2.593	5.579	4.934	6.317

NOTE.—In millions, means of monthly data unadjusted for seasonal variation.

TABLE A6

MANUFACTURING EMPLOYMENT

YEAR	QUARTER I	II	III	IV
1947	15.56	15.35	15.51	15.76
1948	15.64	15.33	15.72	15.64
1949	14.90	14.27	14.38	14.21
1950	14.32	14.75	15.71	16.19
1951	16.33	16.35	16.46	16.44
1952	16.37	16.23	16.58	17.35
1953	17.52	17.64	17.75	17.28
1954	16.58	16.18	16.13	16.37
1955	16.38	16.72	17.07	17.36
1956	17.16	17.13	17.17	17.52
1957	17.28	17.16	17.26	17.00
1958	16.09	15.57	15.92	16.20
1959	16.31	16.72	16.84	16.83
1960	16.94	16.85	16.86	16.54
1961	15.93	16.16	16.54	16.68
1962	16.53	16.82	17.05	17.00
1963	16.69	16.94	17.17	17.18
1964	16.91	17.15	17.50	17.54
1965	17.52	17.87	18.31	18.54
1966	18.60	19.10	19.49	19.66
1967	19.38	19.33	19.46	19.61
1968	19.46	19.69	19.92	20.05
1969	19.93	20.14	20.38	20.23
1970	19.80	19.56	19.43	18.79

NOTE.—In millions, means of monthly data unadjusted for seasonal variation.

TABLE A7

GNP Implicit Price Deflator (1957–59 = 1)

Year	Quarter I	II	III	IV
1947	0.7300	0.7370	0.7490	0.7700
1948	0.7820	0.7920	0.8060	0.8030
1949	0.7970	0.7910	0.7880	0.7890
1950	0.7830	0.7900	0.8080	0.8230
1951	0.8480	0.8540	0.8560	0.8670
1952	0.8670	0.8710	0.8770	0.8830
1953	0.8840	0.8830	0.8840	0.8840
1954	0.8950	0.8960	0.8950	0.8980
1955	0.9020	0.9060	0.9100	0.9160
1956	0.9260	0.9340	0.9460	0.9540
1957	0.9640	0.9710	0.9800	0.9850
1958	0.9930	0.9970	1.0010	1.0060
1959	1.0110	1.0150	1.0190	1.0210
1960	1.0260	1.0310	1.0350	1.0400
1961	1.0430	1.0450	1.0450	1.0510
1962	1.0550	1.0560	1.0580	1.0630
1963	1.0670	1.0700	1.0720	1.0780
1964	1.0820	1.0850	1.0910	1.0960
1965	1.1020	1.1070	1.1100	1.1150
1966	1.1239	1.1349	1.1450	1.1536
1967	1.1617	1.1682	1.1798	1.1935
1968	1.2039	1.2165	1.2290	1.2425
1969	1.2568	1.2722	1.2897	1.3052
1970	1.3257	1.3398	1.3550	1.3739

TABLE A8

Market Yield on 13-Week U.S. Treasury Bills

Year	Quarter I	II	III	IV
1947	0.380	0.380	0.737	0.907
1948	0.990	1.000	1.050	1.140
1949	1.170	1.170	1.043	1.077
1950	1.103	1.153	1.220	1.337
1951	1.367	1.490	1.603	1.610
1952	1.567	1.647	1.783	1.893
1953	1.980	2.153	1.957	1.473
1954	1.060	0.787	0.883	1.017
1955	1.227	1.483	1.857	2.340
1956	2.327	2.567	2.583	3.033
1957	3.100	3.137	3.353	3.303
1958	1.760	0.957	1.680	2.690
1959	2.773	3.000	3.540	4.230
1960	3.873	2.993	2.360	2.307
1961	2.350	2.303	2.303	2.460
1962	2.723	2.713	2.840	2.813
1963	2.907	2.937	3.293	3.497
1964	3.530	3.477	3.497	3.683
1965	3.890	3.873	3.863	4.157
1966	4.603	4.580	5.030	5.200
1967	4.513	3.657	4.293	4.743
1968	5.040	5.513	5.197	5.580
1969	6.087	6.190	7.010	7.347
1970	7.210	6.667	6.327	5.350

NOTE.—Means of daily data.

TABLE A9

Civilian Workers Unemployed

Year	Quarter I	II	III	IV
1948	2.601	2.276	2.237	1.995
1949	3.284	3.586	3.993	3.683
1950	4.584	3.493	2.806	2.273
1951	2.503	1.947	1.921	1.847
1952	2.200	1.883	1.910	1.540
1953	1.975	1.677	1.614	2.074
1954	3.860	3.728	3.527	3.018
1955	3.512	2.915	2.512	2.474
1956	3.084	2.896	2.575	2.450
1957	3.038	2.798	2.624	2.975
1958	4.912	5.040	4.588	3.868
1959	4.558	3.555	3.382	3.463
1960	4.056	3.720	3.637	3.991
1961	5.471	4.957	4.478	3.948
1962	4.475	3.891	3.709	3.573
1963	4.646	4.165	3.788	3.680
1964	4.401	3.937	3.496	3.303
1965	3.938	3.588	3.145	2.794
1966	3.114	3.038	2.791	2.566
1967	3.099	2.917	3.029	2.858
1968	3.097	2.803	2.865	2.501
1969	2.848	2.747	3.003	2.726
1970	3.644	3.868	4.341	4.501

NOTE.—In millions, means of monthly data unadjusted for seasonal variation.

TABLE A10

Civilian Labor Force

Year	Quarter I	II	III	IV
1948	59.06	60.60	61.89	60.94
1949	59.82	61.20	62.33	61.81
1950	60.96	62.34	63.15	62.40
1951	61.05	61.88	62.76	62.39
1952	61.08	62.04	62.85	62.58
1953	62.40	63.01	63.61	63.04
1954	62.80	63.75	64.31	63.72
1955	63.03	64.65	66.20	66.22
1956	65.08	66.81	67.53	66.78
1957	65.58	67.08	67.79	67.25
1958	66.36	68.00	68.45	67.75
1959	66.94	68.62	69.22	68.69
1960	67.67	70.08	70.68	70.07
1961	69.59	70.94	71.14	70.17
1962	69.31	70.85	71.56	70.73
1963	70.32	72.19	72.74	72.09
1964	71.52	73.72	73.95	73.17
1965	72.71	74.85	75.54	74.71
1966	73.83	75.95	76.98	76.35
1967	75.51	77.08	78.70	78.10
1968	77.06	78.92	79.90	79.06
1969	78.87	80.51	82.10	81.45
1970	81.23	82.58	83.82	83.22

NOTE.—In millions, means of monthly data unadjusted for seasonal variation.