MARKETS AND EFFICIENT RESOURCE ALLOCATION IN URBAN AREAS*

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Summary

The purpose of the paper is to put forward a model in which it is possible to analyze the efficiency of market resource allocation in urban areas. The model is a linear programming formulation in which the various activities by which goods can be produced are interpreted to represent production with different land intensities, or buildings of different heights. The urban area produces predetermined amounts of a finite number of goods to be exported from the urban area, and housing for the urban area's workers. The model determines the optimum location of production of each unit of each export good and of housing in the urban area. It also determines the optimum amounts of resources to be devoted to the urban area's transportation system and the optimum amount of congestion.

The paper concludes by a demonstration that correct pricing of the transportation system is the key to efficient resource allocation by markets and it provides a measure of the loss of welfare from improper pricing of transportation facilities.

Introduction

Despite the obvious and great practical importance of the subject, relatively little is known about the efficiency of market resource allocation in a spatial context. Careful formal analysis of the problem dates from an important paper by Koopmans and Beckmann in 1957 [3]. They analyzed a quadratic assignment model in which a fixed number of activities is to be assigned to a fixed number of locations so as to maximize total net revenue from the activities. Net revenue consists of gross revenue, associated with the assignment of particular activities to particular locations, minus the cost of transporting predetermined amounts of output between pairs of activities. They show that competitive markets cannot sustain an efficient assignment of activities to locations in their model.

Models published subsequently tend to be oriented either toward planning or market analysis, but there has been little work in which the two problems are related.

A growing literature, mostly the work of operations researchers, has analysed spatial planning problems, referred to as location-allocation problems. A careful

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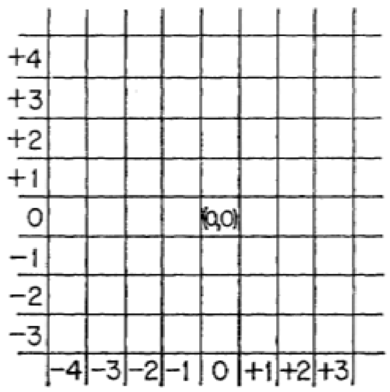
survey has been published by Scott [6]. The common characteristic of such models is that locations must be chosen for a set of activities and then a pattern of shipments among the activities and possibly other locations must be established. Potential locations may be discrete or continuous and a variety of assumptions may be made about alternative origins, destinations and routes of shipments. Most models make strong assumptions about indivisibility of activities. Efficient solutions are difficult to compute in many such models and a variety of heuristic algorithms has been put forward. Both the context and the examples used make it clear that the interests of authors of location-allocation models are in planning rather than market analysis, and most of the spatial relationships are not specifically those of an urban area. Common interpretations are the location of a set of branch factories or warehouses with which a corporation can supply a system of national markets. Most authors do not raise the issue whether market or other decentralized decision making procedures could achieve or sustain an efficient location-allocation pattern.

A second set of models, specifically urban in character, has been formulated to analyze the spatial patterns of activities that result from decentralized markets. Many of these models are descendents of that published by Lowry in 1964 [4]. A recent survey is by Goldner [2]. Such models derive the urban spatial patterns from conditions of supply and demand, profit and utility maximization, and market equilibrium. Most models involve simultaneous relationships and significant nonlinearities. As with the planning models, numerical solution is often difficult and expensive. Whereas the optimization models have no market interpretation, the market-oriented models contain no optimality criteria. Most are either too complex or inadequately articulated to permit determination of whether market solutions are efficient and, if not, what kind of intervention would be best to ensure efficiency. The situation is paradoxical because most of the market-oriented models are formulated to guide public sector planning for future urban growth.

The present paper has two purposes. The first purpose is to further the state of the art of urban model building by showing how models can be constructed that not only are solvable when formulated in realistic detail, but also have both market and planning interpretations. The second purpose is to formulate in detail an urban model designed to shed light on what is widely believed to be one of the most important sources of inefficiency in urban areas, namely improper pricing and resource allocation in the urban transportation system. The model formulated in the next section is designed to make it possible to compute, at least approximately, the loss of welfare from improper pricing and resource allocation in the urban transportation system. Within the framework of the model, it is also possible to show what form of public intervention would enable decentralized markets to allocate resources efficiently. In the final section of the paper, some suggestions are made as to how the model can be modified and generalized to study other possible reasons for inefficiency.



A linear programming model of an efficient urban system

The model developed here is designed to show that urban phenomena often thought to involve important nonlinearities can nevertheless be formulated within a linear programming framework. Specifically, attention is concentrated on congestion and the capital-land substitution by which the intensity of urban land use is determined. The advantages of the linear programming framework are of course that powerful computational techniques are available and that well known theorems about dual solutions provide hints concerning the efficiency of market resource allocation.

The spatial representation of the urban area is shown in the figure. The land surface available for urban use is divided into a rectangular grid. It is assumed that all goods imported into and exported from the urban area pass through a predetermined point, such as a harbor or railhead, and the point is placed at the center of the square designeatd (0.0). This square will become the center of the urban area. The assumption is unrealistic in that large and increasing amounts of goods are shipped by road directly between activities in various suburban locations and points outside the urban area. Although the assumption can be modified, it is not yet clear what modification is most realistic yet does not excessively complicate the model. Numbering the squares as shown, each square can be designated by the ordered pair (i, j), $i, j = \pm 0, 1, 2 \ldots$ The size of a square is of course arbitrary. In empirical work it would depend on the detail desired, data limitations, and the cost and capacity of available computing equipment.

Assumptions will be introduced below that imply that the same kinds and amounts of activities take place in all the squares that are a given travel distance from the city center. Considerable complication is avoided if it is assumed that the transportation system is designed to move traffic only in the north, south, east and west directions and through the center of each square. Then the distance that must be traveled between the center of (i, j) and the city center at the center of (0,0) is

$$d_{ij} = |i| + |j|$$

The obvious alternative assumption is that trips to the city center are made by straight line travel, in which case distance would be $d_{ij}=(i^2+j^2)^{1/2}$. Both assumptions are approximations, but the assumption of straight line travel makes the model much more complex. The number S_u of squares that are u squares from the center is

$$S_u=\begin{cases}1 & u=0\\ 4u & u=1,2\ldots\end{cases}$$

It is assumed that $\bar{r}$ commodities are produced in the city. A predetermined amount $\bar{x}_r$, $r=1, 2, \ldots, \bar{r}-1$, of the rth good must be produced in the city and shipped to the center for export. The $\bar{r}$th good is housing for the city's workers and is not exported.

Inputs in the production of export goods include a single kind each of labor, land and capital. These inputs are supplied competitively to the city. The required workers can be attracted to the city at a fixed wage rate w plus the cost of their housing and commuting. Capital is produced on a national market and is available to the city in unlimited quantity at a fixed rental rate R per unit. As much land as is needed can be obtained by extending the edge of the urban area further from the center. Land is available at a fixed rental rate R_A per square (e.g. its agricultural rental). In what follows, $\bar{u}$ is the maximum number of squares from the edge of the city to the center. It need only be chosen large enough to contain at least as much land as is used for urban purposes.

The technology of production is that each of the $\bar{r}$ outputs can be produced by $\bar{s}$ activities. A set of input-output coefficients a_{qrs} is specific to the sth activity, where a_{qrs} is the amount of input q required per unit of output r using activity s, and

$$q=\begin{cases}1 & \text{labor}\\ 2 & \text{land}\\ 3 & \text{capital}\end{cases}\qquad r=\begin{cases}1,2,\ldots,\bar{r}-1 & \text{export goods}\\ \bar{r} & \text{housing services}\end{cases}\qquad s=1,2,\ldots,\bar{s}\quad \text{activities}$$

The specific interest in this paper is capital-land substitution, which determines the intensity of urban land use. Thus, the sth activity is thought of as the production of one unit of output in an s-story building.[1] Then the land coefficient should be small for large values of s, since less land is required for a 10-story building in which a unit of output can be produced than for a five-story building in which the same output can be produced. Capital coefficients, however, are large for large values of s, since more capital is needed for a ten- than for a five-story building, if each is designed so that a unit of output can be produced in it. Then unit production costs will increase or decrease with

[1] I am indebted to Robert Dennis for this way of representing the technology, which replaces my clumsier formulation in terms of production on specific floors.

building height depending on the precise relationships among the input-output coefficients and on relative factor prices. Labor input-output coefficients may also vary with building height. More labor is needed per unit of output in a tall building because a great deal of time must be spent in vertical commuting. However, the model does not require any specific pattern of labor coefficients for various activities.

This representation of technology permits the introduction of economies or diseconomies from spatial concentration of economic activity within a linear programming framework. Although $\bar{r}$ is given from outside the model, $\bar{s}$ need only be chosen to exceed the number of floors in the tallest building that is economical in the city in question. $\bar{s}$, like $\bar{u}$, need only be chosen so that certain constraints introduced below are not binding. The programming technology employed here is an approximation in that it ignores the incomplete divisibility of buildings. If 10 units of a commodity can be produced in a 5-story building of modest cross-sectional dimensions, it may not be possible to produce one unit of the commodity in a 5-story building that uses one-tenth the capital, land and labor of the larger building. The possibility of sharing walls permits an economy from increasing the number of rooms per floor in small buildings. This causes inaccuracy in the model only if the output of a commodity is small relative to the output produced in a building of minimum efficient cross-sectional dimensions. In such cases, the inaccuracy can be avoided by the addition of integer constraints on certain variables in the programming model.

In addition to land, capital and labor, inputs that are imported from outside the urban area may also be required in the production of the $\bar{r}-1$ export goods. In this model it must be assumed that imported inputs per unit of output of a particular commodity are the same regardless of the activity used. Then all that is required is the interpretation of transportation costs introduced below to include costs of transporting required imported inputs as well as exports.

It is assumed that one unit of housing is needed per worker. Like export goods, it can be produced in buildings with a variety of numbers of stories. Of course, the pattern of input-output coefficients for housing of different numbers of stories need not be the same as those for export goods production. For example, conventions or rules regarding the amound of uncovered land around houses can be incorporated in the input-output coefficients. Export goods produced in the city are assumed neither to be inputs in housing production nor to be consumed by local workers and their families. This assumption, which appears in many urban models in which it seems to be inessential, permits a very simple representation of the transportation pattern in the present model. Goods and services produced in the urban area for local consumption can be included in the model by interpretting housing consumption to include locally produced consumer goods and services. Then the housing input-output coefficients must include input requirements for such local consumption goods and services.

The production variables in the model are amounts of the $\bar{r}$ goods produced by various activities in various parts of the city. $x_{rs}(u)$ represents the output per square of commodity r in s-story buildings u squares from the center. These variables must satisfy the following inequalities:

$$\sum_s \sum_u 4ux_{rs}(u) + \sum_s x_{rs}(o) \geqslant \bar{x}_r \quad r = 1, \ldots, \bar{r}-1 \tag{1}$$

$$\sum_s \sum_u 4ux_{\bar{r}s}(u) + \sum_s x_{\bar{r}s}(o) \geqslant \sum_r \sum_s \sum_u 4ua_{1rs}x_{rs}(u) + \sum_r \sum_s a_{1rs}x_{rs}(o) \tag{2}$$

Unless otherwise indicated, output sums are over integers 1 to $\bar{r}$, activity sums over 1 to $\bar{s}$, input sums over 1 to 3, and location sums over 1 to $\bar{u}$. Inequalities (1) ensure that total production of good r is at least the amount that must be exported. (2) ensures that housing production is at least sufficient to house the city's workers.

Wherever units of export goods are produced, imported inputs must be shipped from the city center to the square where production takes place and export goods must be shipped to the city center for export. Likewise, workers must commute between their homes and places of work. $T_r(u)$, $r = 1, \ldots, \bar{r}-1$, is the number of units of commodity r that must be shipped through each square u squares from the center. Below, the costs of such shipment will be defined to include the cost of shipping the required imported inputs as well as the export good. $T_{\bar{r}}(u)$ is the number of workers who commute through each square u squares from the center. The various commodities and commuters make different demands on the transportation system, and it is assumed that the total demand on the transportation system in each square u squares from the center can be represented by $T(u)$, where

$$T(u) = \sum_r t_r T_r(u) \quad u = 0, 1, \ldots, \bar{u} \tag{3}$$

The numbers t_r depend on the mode or mix of modes employed in the transportation system. At the end of the paper, generalization of the model to include modal choice is discussed, but the present model includes only a single set of coefficients, which pertain to a particular mode or set of modes. Within that framework, there is a choice of units for $T(u)$. For example, it would be possible to put $t_{\bar{r}} = 1$, in which case t_r would represent the demand placed on the system by a one-square shipment of commodity r and its required imported inputs relative to a one-square round trip of a commuter.

Since all units of export goods must be shipped to the city center, all units produced further than u squares from the center must be shipped through one of the squares u squares from the center. The transportation system will be used efficiently only if the same amount of a particular commodity is shipped through each of the squares a given distance from the center. Therefore, we must have

$$T_r(u)=\frac{1}{4u}\left[\bar{x}_r-\sum_{u'=1}^{u-1}\sum_s 4u'x_{rs}(u')-\tfrac{1}{2}\sum_s 4ux_{rs}(u)-\sum_s x_{rs}(o)\right]\quad \begin{matrix}u=1,\ldots,\bar{u}\\ r=1,\ldots,\bar{r}-1\end{matrix}\tag{4}$$

$$T_r(o)=\tfrac{1}{2}[\bar{x}_r-\sum_s x_{rs}(o)]+\tfrac{1}{4}\sum_s x_{rs}(o)\quad r=1,\ldots,\bar{r}-1\tag{5}$$

The square bracket in (4) is the total number of units of good r produced more than u squares from the center. Production in squares u squares from the center is divided by one-half since, on the assumption that production is distributed uniformly through the square, the average unit is shipped half way through the square in which it is produced. There are $4u$ squares that are u squares from the center, so $1/4u$ of the total goes through each square. (5) says that all the units produced outside (0. 0) must be shipped halfway through (0. 0) whereas the average unit produced in (0. 0) is shipped one-fourth of the way through the square.

The assumptions made above imply that workers do not commute away from the city center on their way to work in an efficient spatial pattern in the city. To see this, suppose the contrary. Suppose that a set of workers lived u squares from the center and worked $u'>u$ squares from the center. Then, without changing the activities used in housing or goods production, total costs of the city would be reduced if the workers' residences and their places of work were interchanged. After the interchange, the workers would still commute the same distance, although in the opposite direction, and the distance the goods must be shipped would decrease by $u'-u$ squares. The interchange should continue until either all the goods production or all the workers have been moved, whichever first exhausts the available land. In either case, outward commuting will cease.

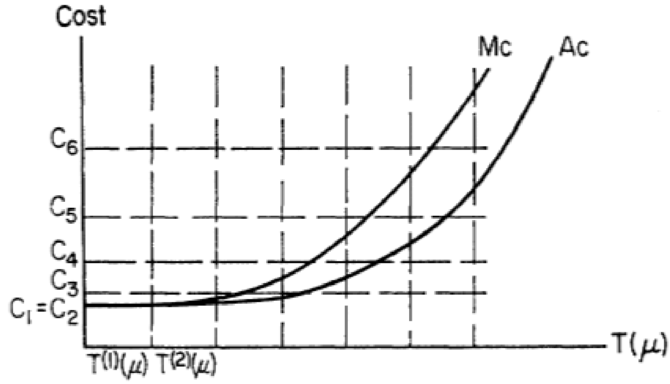
This assumption has the important and realistic implication that residences are more suburbanized than production in an efficient city. It means that $T_{\bar{r}}(u)$ can be written

$$T_{\bar{r}}(u)=\frac{1}{4u}\Bigg[\sum_{u'=1}^{u-1}\sum_s 4u'(\sum_r a_{1rs}x_{rs}(u')-x_{\bar{r}s}(u'))+\sum_s(\sum_r a_{1rs}x_{rs}(o)$$
$$-x_{\bar{r}s}(o))+\tfrac{1}{2}\sum_s 4u(\sum_r a_{1rs}x_{rs}(u)-x_{\bar{r}s}(u))\Bigg]\quad u=1,\ldots,\bar{u}\tag{6}$$

$$T_{\bar{r}}(o)=\tfrac{1}{4}\sum_s(\sum_r a_{1rs}x_{rs}(o)-x_{\bar{r}s}(o))\tag{7}$$

$$T_{\bar{r}}(u)\geqslant 0\quad u=0,1,\ldots,\bar{u}\tag{8}$$

(6) and (7) say that commuting through squares u squares from the center equals the excess of employment over housing in squares closer to the city center. (8) prohibits outward commuting. Explicit non-negativity conditions are unnecessary for $T_r(u)$ in (4), since non-negativity is ensured by the fact that (1) will be satisfied as an equality in an optimum city.



The technology of the transportation system is that the capacity of the transportation network in a given square depends on the resources devoted to transportation in that square, but that varying amounts of traffic can use the system depending on the congestion level that is tolerated. Considerable attention has been devoted to the issue of how travel time and travel cost vary with traffic level in a given transportation system. Although there is considerable disagreement as to the precise form of the function, all writers agree that cost per passenger mile or per ton mile varies with traffic in the way shown in the figure. Until traffic reaches a certain level, which can be referred to as the capacity of the system, travel cost per unit of travel is independent of the traffic volume. At volumes beyond the system's capacity, average speed decreases and travel cost increases, mainly because of increases in time spent traveling. The average cost curve rises at an increasing rate, implying that marginal cost to all travelers of an extra traveler exceeds average cost. Indeed, this fact is the basis of the resource misallocation that results from improper pricing of congestion costs [7].

In this paper the marginal cost function is approximated by a step function, as shown in the figure. Within a square, each congestion level is the same width, which depends on the resources devoted to the transportation system. Otherwise, no specific assumption is made about the shape of the cost function except that $c_k \geqslant c_{k-1}$, $k = 2, ..., \bar{k}$. Specifically, there may be several congestion levels at which no congestion occurs. $\bar{k}$ need only be chosen sufficiently large that the congestion level it represents is not reached. Total transportation cost per square u squares from the center is then

$$c(u) = \sum_k c_k T^{(k)}(u) \quad u = 0, 1, \dots, \bar{u} \tag{9}$$

where $T^{(k)}(u)$ is total traffic at congestion level k per square u squares from the center. (9) can be made as accurate an approximation as is desired by choosing a sufficiently narrow width for each congestion level. $c(u)$ includes vehicle costs and time costs of travel, but not the cost of the fixed capital and land resources devoted to the transportation system. They are included separately in the criterion function.

It is assumed that the capacity of the transportation system at a given congestion level is proportional to the land and capital resources devoted to transportation in each square. Then we can write $b_2 T^{(1)}(u)$ and $b_3 T^{(1)}(u)$ for the land and capital requirements for the transportation system in each square at a distance u from the center. b_2 and b_3 are the land and capital input-output coefficients. The description of the transportation system is completed by the requirements

$$\sum_k T^{(k)}(u) \geqslant T(u) \quad u = 0, 1, \ldots, \bar{u} \tag{10}$$

and

$$T^{(k)}(u) \geqslant T^{(k+1)}(u) \quad \begin{matrix} k = 1, \ldots, \bar{k}-1 \\ u = 0, 1, \ldots, \bar{u} \end{matrix} \tag{11}$$

(10) simply says that enough congestion levels must be used to meet total transportation demand. (11) is necessary to prevent the computer from trying to use relatively high congestion levels without using the first level, to avoid the land and capital requirements of the transportation system. A labor input could be included in transportation, but has been excluded because most labor, at least in systems dominated by the automobile, is supplied by users and its cost is included in (9).

The final constraints that the allocation of resources must satisfy are

$$\sum_r \sum_s a_{2rs} x_{rs}(u) + b_2 T^{(1)}(u) \leqslant 1 \quad u = 0, 1, \ldots, \bar{u} \tag{12}$$

which ensure that land use for production and transportation does not exceed that available in any square.

An efficient allocation of resources in the city is one that minimizes the cost of the required production and transportation. This can be expressed as

$$\left.\begin{aligned} \min Z = R\,[&\sum_u \sum_r \sum_s 4ua_{3rs} x_{rs}(u) + \sum_r \sum_s a_{3rs} x_{rs}(o) \\ &+ b_3 \sum_u 4uT^{(1)}(u) + b_3 T^{(1)}(o)] + R_A [\sum_u \sum_r \sum_s 4ua_{2rs} x_{rs}(u) \\ &+ \sum_r \sum_s a_{2rs} x_{rs}(o) + b_2 \sum_u 4uT^{(1)}(u) + b_2 T^{(1)}(o)] \\ &+ w[\sum_u \sum_r \sum_s 4ua_{1rs} x_{rs}(o) + \sum_r \sum_s a_{1rs} x_{rs}(o)] \\ &+ \sum_u \sum_k 4uc_k T^{(k)}(u) + \sum_k c_k T^{(k)}(o) \end{aligned}\right\} \tag{13}$$

subject to (1)–(12) and nonnegativity of $x_{rs}(u)$ and $T^{(k)}(u)$. If, as an approximation, labor input requirements were assumed to be the same for all activities in the production of a given commodity, the last square bracket in (13) would be a constant and could be dropped.

Minimization of (13) subject to (1)–(12) is of course a conventional linear programming problem and can be solved with commonly available computer programs. The dimensions of the problem depend on the size and complexity of the city and on the detail with which it is represented. Equations (3), (4), (5), (6), (7) and (9) define left hand variables that are computationally inessential. The variables that completely define resource allocation in the city are the $x_{rs}(u)$ and $T^{(k)}(u)$. There are $(\bar{u}+1)(\bar{r}\bar{s}+\bar{k})$ such variables, not including slack variables used for computational purposes. The inequalities these variables must satisfy are (1), (2), (8), (10), (11) and (12). The total number of inequalities is $\bar{r}+(\bar{u}+1)(\bar{k}+2)$, not including non-negativity conditions on the variables.

These dimensions are remarkably modest for what appear to be plausible magnitudes in the system. To illustrate, consider a city similar to one for which some preliminary computations have been undertaken. The city had a population of about one million, or a labor force of about 300 000. Typical U.S. metropolitan areas of that size have an area of about 250 square miles. If each square in the model is one square mile, $\bar{u}$ is about 11. Suppose, as is also realistic in many U.S. cities of that size, that the tallest building in the city is about 15 stories. Suppose, in addition, that there are 10 export goods and five congestion levels. Then the total number of inequalities in the model is less than 100 and the total number of variables is less than 2 000.

Characteristics of an efficient city

The solution of the model provides a complete description of the kinds, amounts and techniques of production and the amount of transportation in each square. For planning purposes, the entire list of variables would be needed. But economists are normally interested in summary statistics which can be used to compare an efficient city with actual cities. Summary statistics are desirable not only because it is hard to grasp the overall pattern implied by many hundreds of variables, but also because such detailed data are not available for comparison with most existing cities.

Only preliminary computations have so far been undertaken with the model, and they will be reported only briefly here. The computations used parameters thought to be realistic for a typical U.S. city of about one million people where reasonably accurate parameter values were easily available. Otherwise, parameter values were permitted to vary over broad ranges. The computations were performed for a city with only one export good.

Two extreme patterns bracket the solutions of the model. At one extreme, employment equals housing production in each square, so there is no commuting. At the other extreme, all the land close to the center is used for production of the export good and transportation, whereas all the land further from the center is used for housing and transportation. The second extreme results in a great deal of commuting, but relatively little goods shipment. The solution went from one extreme to the other in response to relatively small shifts in the capital input-output coefficients and the transportation coefficients t_r. Presumably, there would be a tendency for a mixed location pattern in a city with several export goods.

The pattern of optimum congestion was relatively insensitive to parameter changes. In most solutions, transportation cost per ton- or passenger-mile at the city center was about five times as large as at the edge of the city. Congestion falls off rapidly near the city center, and levels off about half way to the edge of the city. It is clear that a considerable amount of congestion is efficient at least near the centers of large cities.

The best measure of overall intensity of land use is the capital-land ratio. It is very large near the center, representing a concentration of 10–15 story buildings. It falls off rapidly near the center and is nearly constant toward the edge of the city. It is interesting that the linear programming technology can generate a relatively complex pattern of capital-land substitution within the urban area.

Finally, the dual variables corresponding to inequalities (12) in the model can be interpretted as shadow prices or market rental rates of land. At the center of the city, this variable was 50 or 100 times as large as R_A. Again, it fell rapidly near the center and leveled off toward the edge of the city. Of course, the shadow price of land becomes zero in the closest squares to the center that are not completely occupied.

Market resource allocation in the city

The usual results about the relationship between linear programming and market efficiency apply to the model presented in this paper [1, Chs. 13, 14]. Specifically, the solution of the dual problem provides a set of competitive prices which will induce profit maximizing firms to allocate resources efficiently in the urban area. Thus, whether markets allocate resources efficiently in an urban area depends on whether there are institutional, technical or other reasons that prevent realization of the competitive solution.

Probably no one would contend that private firms can construct and price an efficient urban transportation system. The entire system must be integrated and coordinated and a firm with the power to do so would have considerable monopoly control. Thus, public sector intervention is required to construct an optimum transportation system and to set optimum prices for its use. The price charged for the use of the transportation system in each square should be the marginal cost, including congestion cost at the optimum level of congestion, of an extra unit of transportation. Two questions arise here. First, can public officials plan an optimum system and compute efficient prices? Second is there a feasible way to meter the use of the transportation system and collect efficient prices, even if they are known? I have nothing to add to the debate on these questions except the observation that the collection of efficient prices is clearly much easier in a public transit system, such as buses or subways, than in an automobile system.

The view taken here is that if the public sector could plan and price an urban transportation system efficiently, there is nothing in the model that suggests that competition would not induce profit-seeking firms to allocate resources efficiently in the urban area. The basic decision unit in the model is the building and it must of course be planned by a single decision making unit. But there appears to be no reason why competition among building and land owners would not produce efficient resource allocation. In urban areas of even modest size there is enough land at about the same distance from the center that substantial monopoly power seems unlikely. The only exception might be land that is very close to the center, of which there is relatively little. In the U.S., there is little evidence about concentrations of ownership of land very close to city centers.

It is widely believed that urban transportation is pervasively underpriced in the U.S. in that no charges are made for marginal congestion costs. This is almost certainly true of road pricing and is probably true, although to a lesser extent, of public transit pricing. The loss of welfare from inadequate pricing of the transportation system is measured not just by excessive congestion that may result, but also by distortions in the locations of housing and production that may result from attempts to avoid congestion and by distortions in the allocation of resources to the transportation system. For example, many U.S. economists and city planners believe that U.S. cities have decentralized excessively since World War II in part because of attempts to avoid excessive congestion by suburbanization. It should be mentioned that in the special case in which $c_1 = ... = c_{\bar{k}}$ in (9), the transportation system is not subject to congestion and no pricing problem exists.

A modification of the model presented above makes it possible to compute the resource misallocation and loss of welfare resulting from inadequate pricing of the transportation system. The natural assumption to make is that the transportation system is priced at average rather than marginal social cost. In the U.S., the assumption is approximately realistic. Automobile users pay gasoline taxes that cover the cost of resources used in urban road construction, and vehicle users pay vehicle and average time costs of their travel. Public transit users pay fares that approximate their share of right-of-way costs and vehicle costs, and transit riders bear average time costs [5]. The accuracy of these assertions is subject to dispute, but at least average cost pricing is the policy that would be followed to enable the transportation system to pay its way, but without regard for congestion costs.

Average cost transportation prices can be computed in the model as follows. Start with the optimum set of land uses and transportation, as computed from the programming model. Call it the initial pattern of land use and transportation. Then the average cost price of transporting a unit of commodity r in a square at u is

$$p_r(u) = t_r \frac{c^*(u)}{T^*(u)} \quad r = 1, \ldots, \bar{r} \tag{14}$$

plus the cost of the land and capital resources used for transportation in each square at u, similarly allocated among the types of transportation. But these resource costs need not be shown explicitly in (14), since they will not vary in the following iteration. In (14), $c^*(u)$ and $T^*(u)$ are computed from (3) and (9), given the initial pattern of land use and transportation. Then solve the programming problem using, not (13), but a modified criterion function in which transportation is valued at prices $p_r(u)$ instead of at its correct value, and holding constant the allocation of land and capital resources to transportation at their values in the initial pattern. That solution will produce a new pattern of resource allocation and hence new values of $c(u)$ and $T(u)$. These can be used in (14) to compute new values of the $p_r(u)$ and the modified criterion function can be minimized again. If this iterative procedure is repeated, it will presumably converge. If so, the result will be a pattern of resource allocation that is optimum given the improper pricing of the transportation system and given the allocation of land and capital resources to transportation. The resource allocation resulting from the iterative procedure is one that profit seeking firms will be induced to achieve given the transportation pricing. It will be an equilibrium allocation in that, given transportation prices, resources are allocated in the best way possible and it yields the given transportation prices. The loss of welfare resulting from improper transportation pricing is the difference between the value of (13) at the end of the iterative procedure and its value in the efficient pattern of resource allocation. These computations are rather cumbersome and have not yet been performed.

In the iterative procedure, the allocation of land and capital resources to transportation has been held constant at its optimum level. An interesting question to ask is how the public sector might make resource allocation decisions in transportation given the improper pricing scheme. A conjecture is that they might allocate to the transportation system whatever resources fares and user fees could pay for. If so, it would be possible, with further computations, to compute the resulting transportation resource misallocation and loss of welfare. If the transportation system is improperly priced, market land rents will presumably not reflect the opportunity cost of land use for transportation, and an inappropriate land allocation to transportation will result.

Extensions and modifications of the model

There are a number of ways in which the model can be extended or modified to make it more realistic or to include additional problems. Some changes merely complicate the calculations, whereas others require basic reformulation of the model.

The most obvious modification is to fix certain land uses outside the model. As it stands, the model is long run and adjustment times may be extremely long when construction and demolition of buildings are involved. Over realistic planning horizons, many land uses are fixed by historical development in existing cities. The model can easily accomodate existing land uses simply by setting the appropriate $x_{rs}(u)$ and resulting transportation demands as constants and striking out appropriate inequalities. Indeed, only increased computational complexity restrains the introduction of restrictions that are specific to particular squares. The entire model can be reformulated with each square identified rather than grouping all the squares a given distance from the center. The effect on the dimensions of the model is roughly to replace $\bar{u}$ by $1+2\bar{u}(\bar{u}+1)$, the number of squares in the city.

An interesting extension of the model would be the introduction of alternative transportation modes. Specific sets of input-output coefficients are relevant for each mode of urban transportation such as automobiles, trucks, buses and subways. With information about the coefficients, the model could be extended to ascertain the mode or mix of modes that was most efficient for cities of particular sizes and other characteristics.

The model can be extended, in mostly obvious ways, to compute the effects of a variety of public policies such as zoning, real estate taxation and building height limitations.

The conceptually most desirable extension of the model would be to permit produced goods to be inputs in the production of other goods and to be locally consumed, as well as being exported. This would destroy the simple transportation pattern in the model discussed here, and would require more sophisticated formulation. Further complication would be introduced by permitting export goods to be exported from the city directly from various suburban locations, presumably by road, as well as being shipped through the city center.

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