An Exposition of Some Subtleties in the Keynesian System

By

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The introduction of supply conditions into the Keynesian system and their integration with the theory of income and interest rate determination pose one of the more formidable subjects for the student of income theory to master. Since Keynesian theory is often the student's first introduction to income theory, and since it is usually the point of departure for discussions of wealth effects and dichotomies in the pricing process, a simple, compact and generalized exposition might prove useful to the student of the subject. This paper attempts such an exposition although generalization is limited to the four cases in which (a) prices and wages are both flexible, (b) prices are flexible and wages are rigid, (c) wages are flexible and prices are rigid, and (d) prices and wages are both rigid. The distinction between the four cases hinges on whether or not firms are prevented from maximizing profits and whether or not workers are impeded in their pursuit of maximum utility[1].

[1] For a discussion of some of these points the reader is referred to the standard works of J. R. Hicks, "Mr. Keynes and the 'Classics': A Suggested Interpretation", *Econometrica*, Vol. V, Menasha, Wis., 1937, pp. 147 sqq., reprinted in: *Readings in the Theory of Income Distribution*, Selected by a Committee of the American Economic Association, Pref.: Howard S. Ellis, Repr., Blakiston Series of Republished Articles on Economics, Vol. III, Philadelphia and Toronto, 1949, pp. 461 sqq.; Franco Modigliani, "Liquidity Preference and the Theory of Interest and Money", *Econometrica*, Vol. XII, 1944, pp. 45 sqq., reprinted in: *Readings in Monetary Theory*, Selected by a Committee of the American Economic Association, Blakiston Series of Republished Articles on Economics, Vol. V, New York, Philadelphia and Toronto, 1951, pp. 186 sqq.; and Don Patinkin, *Money, Interest, and Prices, An Integration of Monetary and Value Theory*, Evanston, Ill., 1956; see also Harry G. Johnson, "Una versione diagrammatica della teoria dell'interesse dei fondi prestabili", *Rivista inter-*

I. Income and Interest Rate Determination

To develop the more complex apparatus used later we first tread over some well-travelled ground, and develop the version of the Keynesian system made popular by Hicks[1]. The equations are based on two equilibrium conditions:

(1) $I(r) = S(y)$

(where I = real investment, S = real saving, r = the interest rate and y = real income),

(2) $L(r, y) = \bar{M}$

(where L = the demand for money and $\bar{M}$ = the stock of money). In this simple version it is assumed that the price level is fixed. Equation (1) states that investment equals saving and gives the relation between r and y for output to be demanded. Equation (2) states that the demand for money equals the supply of money; for diagramatical purposes, however, it will be convenient to split L into its transaction (L^T) and asset (L^A) components. The supply of money is assumed to be constant.

1. The IS Schedule

Equation (1) is developed in Figure 1. Quadrant IV graphs the propensity to save, $S = S(y)$, Quadrant II graphs the incentive to invest, $I = I(r)$, and Quadrant III depicts the condition of equilibrium, I = S. From these relations the IS schedule, $I(r) = S(y)$, can be derived in Quadrant I. For example, at the interest rate $r = r_1$ investment is $I = I_1$. For saving to equal investment $S = S_1$ is necessary. But $S = S_1$ only if income $y = y_1$. Hence, the income level $y = y_1$ must be associated with the interest rate $r = r_1$ in order that investment equal saving. The point Z_1 is therefore one point on the IS schedule. In a similar fashion every other point along IS can be established.

[1] Hicks, *op. cit.* — More generally, one could allow for wealth effects, a marginal propensity to invest, a responsiveness of saving to the interest rate, and some interest elasticity in the supply of money. But the present version is more in keeping with the spirit of the *General Theory*. It should also be mentioned that I ignore throughout the extreme cases associated with the "liquidity trap" (the situation in which interest rates are so low that the demand for money is supposed to be completely elastic) and full employment saving-investment equality at a negative rate of interest.

Figure 1
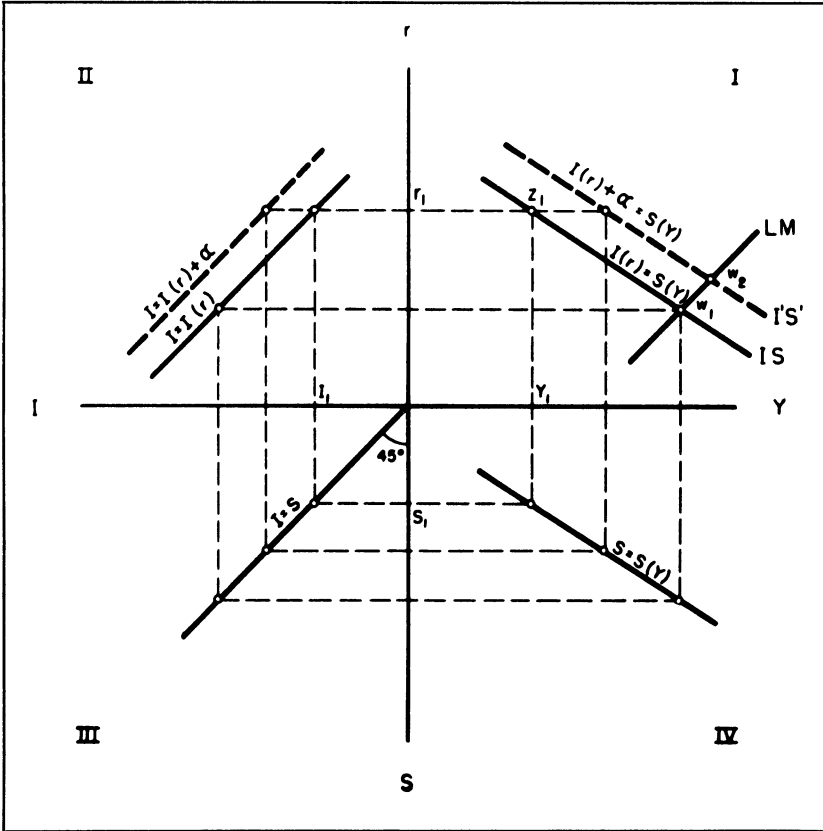
Note that IS is steeper the steeper is I(r) and the greater is the marginal propensity to save S'(y). Note also that an upward shift in investment or a reduction in the propensity to save shifts IS upward and to the right. For example, an increase of investment of the amount α shifts IS to I'S'.

2. The LM Schedule

Equation (2) is developed in Figure 2. In Quadrant IV the line $L^T = L^T(y)$ plots the *transactions* demand for money which is a function of income, while in Quadrant II the line $L^A = L^A(r)$ plots the *asset* demand for money, which is a function of the interest rate. Quadrant III gives the equilibrium condition that the supply of money, $\overline{M}_1$, must equal the sum of the demand for money for both asset and transactions purposes, $L^T + L^A$. The LM schedule can easily be derived from these relations.

For example, consider the interest rate $r = r_1$. At this interest rate the public wants to hold L_1^A of money for asset purposes and this leaves L_1^T of money left over from the given money stock for transactions purposes. But the public will want to hold L_1^T of money only if income is $y = y_1$. Therefore, the interest rate $r = r_1$ must be associated with the income level $y = y_1$ if the demand for money is to equal the supply of money. The point Q is then one point on the LM schedule, and all other points can similarly be derived.

Note that LM is flatter the less steep is $L^A(r)$ and the smaller is the marginal propensity to hold money for transactions purposes. It can also be shown that an increase in the money supply (which can be analyzed by shifting the $\overline{M}_1\overline{M}_1$ line to $\overline{M}_2\overline{M}_2$ in Quadrant III), or a reduction in the demand for money for either transactions or asset purposes, both work to shift LM down and to the right. For example, an increase in the money supply from $\overline{M}_1$ to $\overline{M}_2$ shifts LM from $L\overline{M}_1$ to $L\overline{M}_2$.

Figure 2
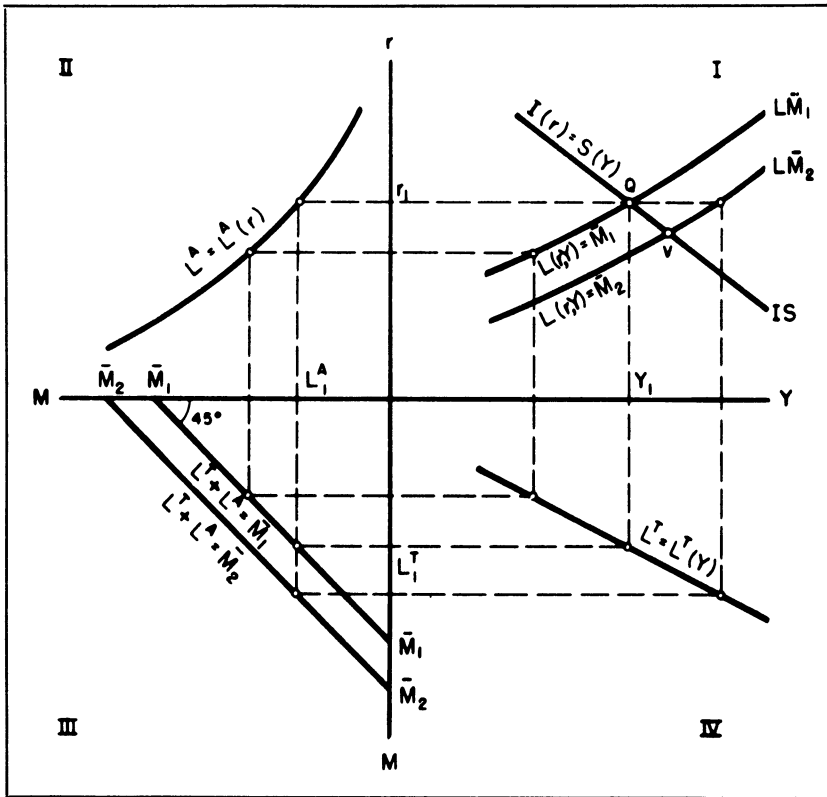
3. The Hicksian Diagram

Combining the two schedules in Quadrants I of Figures 1 and 2 gives a version of the Hicksian diagram. Thus, an increase in the incentive to invest (or a decrease in the propensity to save) will raise both income and the interest rate as equilibrium moves in Figure 1 from W_1 to W_2, or an increase in the supply of money (or a decrease in the demand for money) lowers the interest rate and raises income, as equilibrium moves in Figure 2 from Q to V.

II. Supply Conditions

The preceding analysis is unsophisticated because it does not explicitly account for supply conditions and because the price level and wage are assumed to be constant. It is not immediately obvious how important these assumptions are. A more complete system is necessary and it requires additional equations. If wages and prices are flexible the system is as follows:

(3) $I(r) = S(y)$ (Commodity Market)

This equation is the same as (1) because I, S, and y are all defined in real terms and are not, therefore affected by the price level[1].

(4) $L(r, y) = \bar{M}/p$ (Money Market)

The *real* demand for money equals the *real* supply of money, with p = price level.

(5) $y = \varnothing(n)$ (Production Function)

This is the production function with one variable factor (n = employed labor) which implies $\varnothing'(n) > 0$, and $\varnothing''(n) < 0$ if there is diminishing returns.

(6,7) $\varnothing'(n) = w/p = \psi'(n)$ (Maximum Profit and Utility Conditions)

Firms are maximizing profits only if the marginal productivity of labor equals the real wage rate, w/p, where w = money wages; and labor is maximizing utility only if the real wage rate equals the marginal disutility of labor $\psi'(n)$[2].

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[1] Implicit in (3) is the assumption that expenditure is independent of the distribution of income.

[2] Alternatively, equations (6—7) can be written as

$$(6a, 7a)\ \eta^d\left(\frac{w}{p}\right) = \eta = \eta^s\left(\frac{w}{p}\right)$$

where η^d represents the demand for labor and η^s represents the supply of labor. However (6, 7) is slightly preferable for present purposes insofar as the schedules, in certain cases of rigidities, are not precisely demand and supply curves; there are discontinuities.

The following points are worth noting:

The five equations (3—7) suffice to determine the five unknowns r, y, p, w and n, given the nominal supply of money ($\overline{M}$). However, equations (6,7) alone suffice to determine both employment and the real wage rate, implying that these are independent of the quantity of money. Moreover, the real magnitudes of the system, w/p, n, y and r, can be determined without the aid of equation (4), which implies that the money equation simply determines absolute prices and wages and that changes in the quantity of money leaves unaltered the level of output and the interest rate.

The system can be analyzed graphically by deriving two schedules, EE and RR, relating the interest rate and the real wage rate. These schedules are shown in Quadrant I of Figure 3, and are derived on the basis of the schedules in the other quadrants. Quadrant II plots the IS schedule, equation (3); Quadrant III plots the production function, equation (5); and Quadrant IV plots the labor supply and demand conditions, equations (6,7). For the moment ignore the LM schedule, equation (4).

To derive RR, consider the interest rate $r = r_1$. At this interest rate the only level of output that will clear the market is $y = y_1$ in Quadrant II. To produce this output requires $n = n_1$ of labor (Quadrant III). But n_1 of labor will only be *demanded* if the real wage rate is $(w/p)_1$. Hence r_1 and $(w/p)_1$ are the coordinates of one point on RR. Another point is given by the coordinates of r_0 and $(w/p)_0$ so RR has a positive slope.

To derive EE, consider another interest rate $r = r_2$. At this interest rate the level of output at which saving equals investment is $y = y_2$, which requires $n = n_2$ of employment. But this amount of labor will only be *supplied* if the real wage rate is $(w/p)_2$. Hence r_2 and $(w/p)_2$ are the coordinates of a point on EE. Another point is given by r_0 and $(w/p)_0$ so that EE has a negative slope in that range of the labor supply curve which has a normal slope.

The two schedules have precise interpretations. EE is the relation between the interest rate and the real wage rate at which *the employment levels which maximize labor's utility will produce outputs which can all be sold*. RR, on the other hand, is the relation between the interest rate

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(1) Edwards, *op. cit.*, has pointed out that the Keynesian case does not depend on money illusion on the part of labor; it is sufficient that labor *bargains* for a money wage.

(2) With one variable factor the condition (6) is equivalent to the condition that firms equate marginal cost and price since $w/\varnothing'(n)$ is marginal cost.

(3) The supply curve of labor may bend backward at some (high) real wage.

(4) In the following discussion I shall refer to any discrepancy between price and marginal cost as a "failure of profit maximization," and to any gap between the real wage and the marginal disutility of labor as a "failure of utility maximization," though this is a restricted use of the term "maximization."

Figure 3
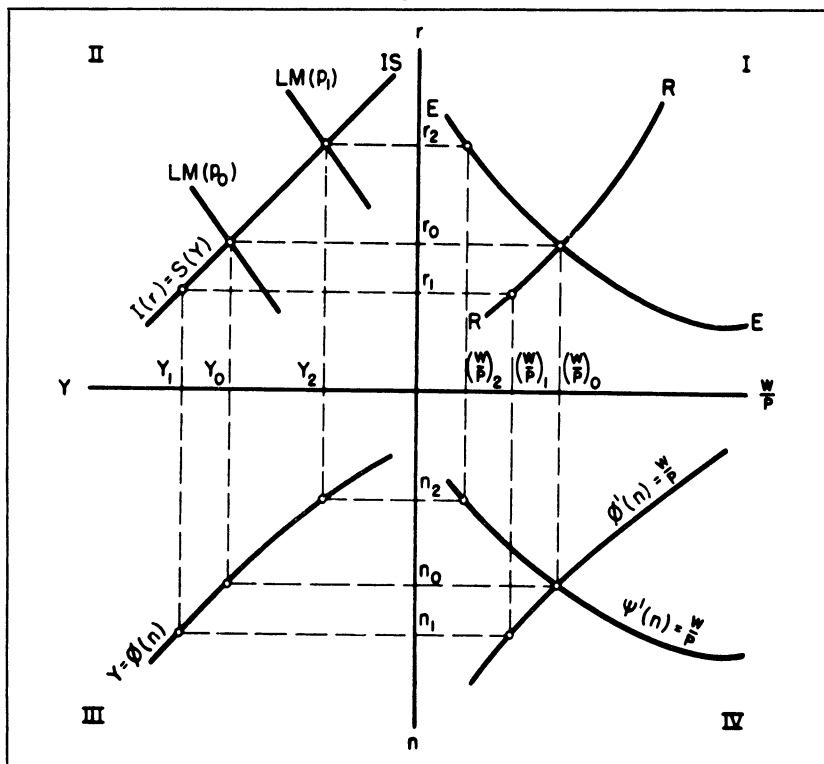
and the real wage rate at which *the employment levels implied by profit maximization (under competition) will yield outputs which can all be sold.* On both EE and RR, therefore, the implied output can be sold, but only along EE is labor maximizing utility, and only along RR are firms maximizing profits. And only at the interest rate $r = r_0$ and the real wage $(w/p) = (w/p)_0$ are *both* utility maximization of labor and profit maximization of firms consistent with production which can exactly be sold.

The system will gravitate to the equilibrium interest rate r_0 and the equilibrium real wage rate $(w/p)_0$ whatever is the stock of money. This can be seen by noting that LM(p) (which shifts with the price level in view of (4)) can be inserted into Quadrant II. Suppose that, at the absolute price level, $p = p_1$, LM intersects IS at the interest rate $r = r_2$ and the output level $y = y_2$. To produce this output $n = n_2$ of labor is required. But at this employment level the demand price for labor is higher than the supply price. At the supply price of labor $(w/p)_2$ firms

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will want to produce an output greater than y_2 and will want to hire
more labor than the amount $n = n_2$ forthcoming at that supply price.
Hence money wages will rise to equilibrate the labor market and the
price level will fall to equilibrate the commodity market. As the price
level falls, the real value of money balances increases and the LM schedule
shifts downward. Only when LM intersects IS at the interest rate $r = r_0$
and the output level $y = y_0$ will the system be in equilibrium. There is
only one price level at which this position of LM is possible, the price
level $p = p_0$, which is the equilibrium price level.

III. Rigidities in the System

The EE and RR schedules can now be used to interpret price and
wage rigidities. When there are price or wage rigidities the real values

Figure 4
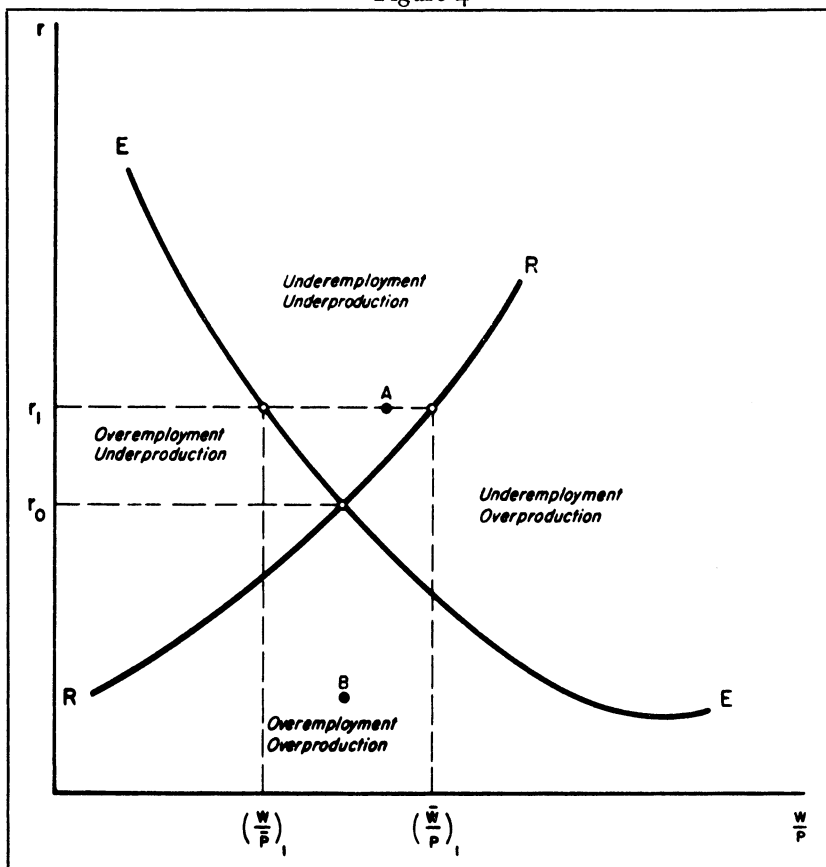
of the system are no longer independent of the money supply. The authorities can alter the interest rate, the real wage rate, the level of employment and the level of output by adjusting the supply of money.

Consider Figure 4, which reproduces the EE and RR lines of Figure 3. As we have already noted, along EE workers, for any given wage rate, are offering the utility-maximizing employment and the ensuing output can all be sold at the corresponding interest rate; there is full employment. But above and to the right of EE the marginal disutility of labor is less than the real wage rate and there is involuntary unemployment, while below and to the left of EE there is involuntary overemployment.

Similarly (and as already noted), along RR the profit-maximizing output can all be sold. On the other hand, above and to the left of RR the marginal productivity of labor is greater than the real wage rate, which implies that price is greater than marginal costs (hence there is "underproduction") while below and to the right of RR price is less than marginal cost (hence there is "overproduction").

1. Rigid Money Wages and Flexible Prices

If the money wage rate is pegged the system (3—7) contains five equations in only four unknowns, r, y, n and p. It is therefore over-determined and this means that one of the equations cannot be satisfied. Since a change in *money* wage rates is the instrument by means of which labor brings its marginal disutility of work into line with the real wage rate, the fixing of money wages means that equation (7), $\overline{w}/p = \psi'(n)$, will be frustrated. The system then is

(8) $I(r)\quad = S(y)$ Commodity Market
(9) $L(r,y) = M/p$ Money Market
(10) $y\qquad = \varnothing(n)$ Production Function
(11) $\varnothing'(n) = \overline{w}/p$ Maximum Profit Condition

which is completely determined.

Consider, for example, the interest rate $r = r_1$ which, for simplicity, we can suppose to be maintained by the Central Bank by appropriate adjustments in the money supply. At this interest rate there is only one level of output at which supply will equal demand in the goods market, and there is, therefore, only one equilibrium level of employment. Now it should be clear that at this level of employment the real wage rate which is equal to the marginal disutility of labor must differ from the real wage rate which is equal to the marginal productivity of labor; this must be true at every interest rate except the equilibrium interest rate $r = r_0$. But the conflict will be resolved in favor of profit maximization of firms rather than utility maximization of labor, because output will

respond whenever price differs from marginal cost (which is the same as marginal productivity differing from the real wage), and the change in output will immediately occasion an adjustment in the price level, altering the real wage. At the point A, for example, the real wage is less than the marginal productivity of labor, so firms attempt to increase employment and output, but this simply bids down the price level until the real wage has risen to a point on RR, corresponding to the real wage $(\overline{w}/p)_1$. Similarly, at points in the graph to the right of the RR line the real wage will tend to move downward.

With both wages and prices flexible, it will be remembered, the nominal quantity of money did not make any difference to the real equilibrium of the system. With money wages inflexible, however, one degree of freedom is lost; the money supply changes the equilibrium of the system. There is, therefore, only one level of the money supply which will yield a full employment equilibrium, given the level of wages, and that is the money supply necessary to effect the rate of interest $r = r_0$. At the point A, of course, there is general underemployment and underproduction.

2. Flexible Wages and Rigid Prices

Price rigidity may arise from monopoly or cartel arrangements or government control. If the price level is rigid but the wage rate is flexible, the general system of equations is again over-determined and one of them will be left unsatisfied. Since labor will always be willing to lower or raise money wages according to whether the marginal disutility of work exceeds or falls short of the going real wage rate, this equation must remain, and it is the marginal productivity equation which must be left unsatisfied. The system is then

(12) $I(r)\ \ \ = S(y)$ Commodity Market
(13) $L(r,y) = \overline{M}/\overline{p}$ Money Market
(14) $y\ \ \ \ \ = \varnothing(n)$ Production Function
(15) $w/p\ \ = \psi'(n)$ Maximization of Labor Utility

which is completely determined.

Consider again the interest rate $r = r_1$. Again, the level of output and employment at which investment is equal to saving will involve a difference between the real wage at which labor maximizes utility and the real wage at which firms maximize profits. But as long as money wage rates are flexible, the level of money wages and hence (with prices fixed) real wages will fall until the marginal disutility of labor is equal to the real wage rate. There is no provision for profit maximization of firms in the usual sense. At the interest rate $r = r_1$ the equilibrium real wage is $(W/\overline{p})_1$ which is a real wage consistent with full employment.

The implication that firms do not maximize profits seems at first puzzling. In the position under consideration in Figure 4 price exceeds marginal cost. Why is it that firms do not expand?

If firms expanded employment and output, the additional supply could not be sold at the given interest rate. Inventories would simply pile up. Thus, the additional output yields zero marginal revenue to firms. To put it another way, while the value of the marginal product exceeds the real wage, the marginal revenue product is less than the real wage — it is zero.

3. Rigid Wages and Rigid Prices

This means that two degrees of freedom are lost from the system (3—7) and that two equations cannot be satisfied. In this case both equations (6,7) must be dropped and the system becomes

(16) $I(r) \quad = S(y) \quad$ Commodity Market
(17) $L(r,y) = M/p \quad$ Money Market
(18) $y \quad = \varnothing(n) \quad$ Production Function

which suffices to determine r, y and n. The real wage is of course given once the price level and the wage rate are specified.

In Figure 4 any point on the diagram is a possible situation. Given the interest rate and the real wage the nature of the disequilibrium in the factor market is revealed. Thus, at the interest rate and real wage implied by the point B, price exceeds marginal cost and the marginal disutility of labor is less than the real wage rate. There is no mechanism, as in the other cases[1], for ensuring either full employment (a point on EE) or profit maximization (a point on RR).

IV. Conclusions

The four cases can be summarized as follows:

(1) Flexible wages and prices imply profit maximization of firms and utility maximization of labor. The system settles at a full employment equilibrium in which the real values of the variables are independent of the quantity of money.

(2) Any rigidity in the system means that the interest rate, income, employment and the real wage rate are all affected by changes in the quantity of money. If money wages are rigid and prices are flexible, firms

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[1] In addition to these cases, the reader may wish to experiment with situations in which there is excess supply of or excess demand for goods, i.e., situations where investment does not equal saving; or with situations in which money demand does not equal money supply. Those situations are, of course, more likely in the short run than in the intermediate run under consideration.

can maximize profits but there is full employment only at a unique level of the quantity of money.

(3) If prices are rigid and money wages are flexible, there is automatic full employment, but firms produce at outputs where marginal cost differs from the fixed price, except when the supply of money is at a unique equilibrium level.

(4) If both wages and prices are fixed, the general case is one where the marginal disutility of labor differs from the real wage, and marginal cost differs from the fixed price level[1].

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Zusammenfassung: Erklärung einiger Feinheiten im Keynesianischen System. — Angebotsbedingungen werden in das Keynesianische System mit Hilfe von Vier-Quadranten-Schaubildern eingeführt, um die Erklärung zu vereinfachen und zu verallgemeinern. Die Verallgemeinerung umfaßt die vier Fälle, in denen a. sowohl Preise als auch Löhne beweglich, b. Preise beweglich und Löhne starr, c. Preise starr und Löhne beweglich und d. sowohl Preise als auch Löhne starr sind. Die Unterscheidung in jedem einzelnen Fall wird davon abhängig gemacht, ob die Unternehmungen verhindert werden oder nicht, bis zu dem Punkt zu produzieren, an dem die Grenzkosten dem Preis gleichen, und ob die Arbeiter daran gehindert werden oder nicht, ihre Dienstleistungen bis zu dem Punkt zur Verfügung zu stellen, an dem der Grenznutzen der Arbeit dem realen Lohnsatz gleicht. Die Analyse schließt mit einer Diskussion der Beziehungen zwischen dem Zinssatz und dem realen Lohnsatz, bei dem verschiedene Ungleichgewichtssituationen (Überbeschäftigung, Unterbeschäftigung, Überproduktion, Unterproduktion) überwiegen werden.

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Résumé: Une exposition de quelques finesses du système Keynes. — Les conditions d'offerte sont introduites dans le système Keynes à l'aide de diagrammes à quatre quadrants, afin de simplifier et de généraliser l'exposition. La généralisation

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[1] This article was expository, rather than critical, but the reader is entitled to a reminder about some of the defects in the Keynesian system: (1) The savings function and the liquidity function cannot plausibly be separated completely since excess or deficient liquidity will itself affect spending. (2) The investment relation I(r) has a downward slope because interest rate changes alter the demand price for the stock of capital goods and hence the flow supply; but relative changes in the production mix of consumer and capital goods may also affect the marginal productivity of capital and hence the *position* of I(r), just as changes in employment, as D. Meiselman has observed, will affect the ratio of capital and labor and thus capital's marginal productivity. (3) The LM-IS configuration relates *stock* equilibrium of money and *flow* equilibrium of goods so that the intersection of the schedules gives equilibrium only if the rate of expansion of the monetary stock is not constant but instead equals the rate of growth; otherwise, as I show in a forthcoming paper in the *Journal of Political Economy*, there will be a gap at the point where the schedules intersect between the cost of holding money and the real rate of interest. See my "Inflation and Real Interest", *The Journal of Political Economy* (Chicago, Ill., June 1963) for a discussion of inflationary equilibrium in a full employment economy.

comprend les quatre cas suivants, où a. les prix aussi bien que les salaires sont flexibles, b. les prix sont flexibles, les salaires rigides, c. les prix sont rigides, les salaires flexibles, d. prix et salaires sont rigides. La distinction dans chaque cas dépend de si, oui ou non, les entreprises sont empêchées de produire jusqu'au point où le coût marginal égale le prix, et si, oui ou non, les ouvriers sont empêchés de vendre leurs services jusqu'au point où l'utilité marginale du travail égale le vrai taux du salaire réel. L'analyse se termine avec une discussion des relations entre le taux d'intérêt et le taux du salaire réel, où différentes situations de déséquilibre (suremploi, sousemploi, surproduction, sousproduction) se produiront.

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Resumen: Una exposición de unas finezas del sistema Keynesiano. — Las condiciones de oferta son introducidas en el sistema Keynesiano por medio de diagramas a cuatro cuadrantes para simplificar y generalizar la exposición. La generalización comprende los cuatro casos siguientes en los cuales a) los precios así como los salarios son flexibles, b) los precios son flexibles, pero los salarios son rígidos, c) los precios son rígidos y los salarios son flexibles y d) los precios y los salarios son rígidos. La distinción en cada uno de los casos depende de lo, si las empresas son impedidas — sí o no — producir hasta el punto en el cual el coste marginal es igual al precio, y si los obreros son impedidos — sí o no — vender sus servicios hasta el punto en el cual la utilidad marginal del trabajo es igual al tipo del salario real. El análisis termina con una discusión de las relaciones entre el tipo de interés y el tipo del salario real en el cual prevalecerán diferentes situaciones de desequilibrio (superempleo, subempleo, superproducción, subproducción).

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Riassunto: Un'esposizione di alcune finezze del sistema Keynesiano. — Le condizioni di offerta sono introdotte nel sistema Keynesiano per mezzo di diagrammi a quattro quadranti onde semplificare e generalizzare l'esposizione. La generalizzazione comprende i seguenti quattro casi nei quali a) i prezzi com'anche i salari sono flessibili, b) i prezzi sono flessibili, però i salari sono rigidi, c) i prezzi sono rigidi ed i salari sono flessibili e d) i prezzi ed i salari sono rigidi. La distinzione in ciascuno dei casi dipende da ciò, se le imprese sono impedite — sì o no — di produrre fino al punto nel quale il costo marginale è uguale al prezzo, e se i lavoratori sono imprediti — sì o no — di vendere il loro servizi fino al punto nel quale l'utilità marginale del lavoro è uguale al tasso del salario reale. L'analisi finisce con una discussione delle relazioni fra il tasso di interesse e il tasso del salario reale nel quale prevaleranno differenti situazioni di squilibrio (sopraoccupazione, sottooccupazione, sopraproduzione, sottoproduzione).