General equilibrium in a simple economy with imperfect information

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Abstract. We determine equilibrium in a simple economy with imperfect information. Each risk-neutral household acts as a firm, making wage offers according to an endogenous wage distribution, and labourer, sampling a stochastic number of offers made by other households over its search period. This economy with imperfect information and decentralized production and consumption will not maximize welfare. Policy must be carefully formulated because there are two non-trivial equilibria; an unstable Keynesian underemployment equilibrium with coordination failure resulting in underemployment and reduced search effort; and a stable high-effort, low-unemployment equilibrium, reminiscent of a Classical equilibrium with frictional unemployment.

Equilibre général dans une économie simple où l'information est imparfaite. L'auteur définit l'équilibre dans une économie simple où l'information est imparfaite. Chaque ménage (qui est neutre face au risque) agit comme une firme et fait des offres de salaires selon une distribution de salaires endogène, alors que le travailleur procède à échantillonnage d'un nombre stochastique d'offres faites par d'autres ménages au long de la période de recherche. Cette économie où l'information est imparfaite et la production et la consommation sont décentralisées ne maximise pas le niveau de bien-être. La politique publique doit être formulées avec beaucoup de soins parce qu'il existe deux équilibres non-triviaux: un équilibre Keynesien de sous-emploi qui est instable où le manque de coordination résulte en un niveau de sous-emploi et en un effort de recherche d'emploi réduit, et un équilibre stable où l'effort de recherche d'emploi est élevé et le taux de chômage bas qui n'est pas sans rappeler l'équilibre classique avec chômage frictionnel.

I. INTRODUCTION

Modern interpretations of the Keynesian explanation for an underemployment equi-

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librium rest upon a breakdown in coordination.[1] In a seminal paper, Diamond (1982) modelled such failures as arising from ex ante differences in production costs across firms and matching (search) costs among labourers, resulting in a multiplicity of equilibria, some with Keynesian underemployment properties. Diamond does not explicitly model the search technology, however, and he is thus unable to describe fully the role of costly information. In a separate literature, search theorists have modelled the interaction between information, wages, and employment in partial equilibrium models of the labour market. We present a general equilibrium model of rational and ex ante identical agents facing non-trivial search costs who stochastically match production with consumption. These search costs prevent information from becoming perfect, while stochastic matching causes an ex post asymmetry in returns to households' search effort. By deriving the static general equilibrium between labourers and firms under imperfect information, we provide insights into the search externalities that can arise, yielding a trivial, zero-output equilibrium and two non-trivial equilibria: one an unstable low-output equilibrium, and the other a stable, high-output equilibrium.

In a seminal article relating seemingly disjoint literatures on non-competitive macroeconomic equilibria, Cooper and John (1988) coin the term 'strategic complementarity' for interactions between agents in a macroeconomy. Search externalities are a subset of these interactions. The authors model equilibria that result when agents' pay-off functions and strategy sets are interrelated. In a Nash equilibrium with such strategic complementarities arising from search externalities, unemployment or overemployment of resources can occur. For instance, in Diamond's (1982) model, multiple equilibria occur as a result of trading externalities. He offers a simple story: a labourer scans the beach looking for a coconut tree to harvest, where trees differ in their harvest costs. Once the labourer has harvested his bunch, he must search for someone with whom to trade. The labourer's belief in his ultimate success in finding such a trading partner will affect his willingness to climb the coconut tree and hence output and employment. If production costs differ, a rise in the likelihood of a match between production and consumption permits additional production opportunities and influences trade and employment. However, Diamond adopts a rigid search technology in which the matching rate between agents is parametric, while the absolute number of matches depends on the number of suppliers and demanders for the traded commodity. We develop a more realistic model of search behaviour in a general equilibrium model for a simple stochastic and costly information economy. We assume that identical households own both capital and labour and simultaneously hire labourers (act as firms) and offer their labour services (act as labourers) in each period to maximize utility. Like Diamond (1982), we ignore the perfect information strategy in which each agent acts as both producer and consumer and trades with itself or a long-term contracted

1 See Leijonhufvud (1981) for such interpretations to the Keynesian model. The literature has traditionally modelled coordination failures as arising from agent asymmetries. Examples include differences in efficiency wage-setting properties across industries (Kahn and Mookherjee 1988) or less specified matching delays among suppliers and demanders in factor markets (Hosios 1990).

partner.[2] As in Wilde's (1977) model of labour search under imperfect information, we assume non-sequential stochastic search in which labourers devote a certain ex ante effort to search each period, which yields an average expected number of wage offers that arrive according to a poisson process.[3] In the resulting symmetric general equilibrium in wage offers and labourers' search strategies, profits arising from firms' wage setting are dissipated through wage dispersion, unemployment, and excess capacity.[4] We determine the equilibrium average wage, employment rate, and return to capital in this symmetric equilibrium. We also explore the sensitivity of the unemployment rate to changes in the productivity of labour, search costs, and the value of leisure. Rather than the multiplicity of equilibria postulated by Diamond, we find two distinct non-trivial equilibria revealing two different levels of unemployment and output.[5]

We present the model in the second section of the paper. In section III we characterize firms' symmetric offered wage distribution given the distribution of labourers' common search effort and reservation wage. In section IV we determine the optimal search strategy of labourers given the average wage offered by firms. We derive the equilibrium average market wage and employment rate in section V and explore their sensitivity to changes in exogenous parameters in section VI. We summarize our results in section VII. For the reader's convenience, we denote by an asterisk those parameters we take as exogenous and provide a notational glossary in an appendix.

II. DESCRIPTION OF THE MODEL

Consistent with Diamond's analogy, we assume that households own both a coconut tree and a unit of labour but cannot 'work' their own tree. Unlike Diamond, however, we introduce no ex ante production asymmetries. Instead, we assume a fixed

2 While models of long-term contracting between labourers and firms reduce amortized information costs and avoid this coordination problem, such models are unable to explain Keynesian underemployment equilibrium.

3 Non-sequential search would occur if the search process requires some time precommitment, represented perhaps by a leave from a present job or the entitlement period of unemployment insurance. Morgan (1983) notes that the optimal strategy may then be a series of non-sequential searches over a sequence of periods.

4 There may also exist asymmetric equilibria. We concentrate only on symmetric equilibria in this analysis.

5 Of the two equilibria, the underemployment equilibrium is found to be unstable, while the high-wage, low-unemployment solution is stable to perturbations in the equilibrium search intensity. While we do not specify a time-dynamic model here, Howitt and McAfee (1988) explore the stability of the stationary state in macroeconomic models with multiple equilibria. They note that an equilibrium in a static model, which is argued to represent a stationary state in a more fully specified dynamic version, may be unstable in the static version but locally stable in its dynamic extension. This occurs if there is a rise in the 'marginal adjustment cost faced by a firm trying to expand its activity level' to keep pace with increased activity by other firms. For instance, Diamond's unstable lower equilibrium point may be stable in a dynamic model of matching. Similarly, the lower equilibrium point we derive may be stable in a dynamic extension of our model if high output firms can attract more available labourers (perhaps through matching economies of scale).

coefficient constant returns to scale technology with an exogenous marginal and average product of labour y^* per unit of capital. In the spirit of a simple economy, all coconut trees and labourers are identical, and each risk-neutral household owns one unit of output (one palm tree) and one unit of labour. Acting as a firm, each household offers wages according to a mixed offered-wage strategy, and it earns, on average, a market-determined return from the surplus of production over wages paid in each period. Each household is simultaneously a labourer, selling its unit of labour to the sampled firm offering the highest wage in excess of a common reservation wage α^*. Firms are conferred a limited degree of monopsony power in their offered wage strategies, because labourers cannot be perfectly informed when information arrives stochastically. In the symmetric Nash equilibrium, firms earn an identical (endogenous) average return to capital r while searching labourers determine their common optimal search intensity to maximize expected income.[6]

Each labourer pays a sunk per period cost $C(\mu)$ to sample a stochastic expected number of wage offers μ. Such costs represent precommitted costs of search that rise linearly with the expected number of samples. We assume that this search technology exhibits constant expected marginal sampling costs c^* in the average number of job offers μ, resulting in an ex ante committed cost $c^*\mu$ of sampling an expected number of μ wage offers.[7] Then expected income is given by

$$Y^e(\mu) = r + E(\mu)w_L + U(\mu)\alpha^* - c^*\mu,$$

where r is the market-determined return to each household's unit of capital, $E(\mu)$ is the employment rate as a function of search intensity μ, and $U(\mu)$ is the unemployment rate $(1 - E(\mu))$. Wage-taking labourers maximize expected income through their choice of search intensity μ, given their expectation w_L of the average market wage, their exogenous value of leisure α^*, and marginal search costs c^*.

Labourers are differentiated ex post only by the number of offers they sample before choosing the job offering the highest wage. Because there is the same number of firms as labourers in this simple economy, the search rate μ represents the average number of offers sampled by each labourer and the average number of wage offers made by each firm.[8] If offers arrive stochastically and evenly over the search interval, the offer arrival rate can be described by the poisson process. An investment $c^*\mu$ in job search then yields a stochastic, mean under μ job offers, giving the labourer a positive probability $p_i(\mu)$ equal to $\mu^i \exp(-\mu)/i!$ of receiving

6 Note that in the traditional competitive simple economy, a Walrasian auctioneer ensures that no trades are made until the market clears. Hence, one unit of capital is matched with precisely one labourer in each period. However, this pattern cannot occur given stochastic matching.

7 This is in the spirit of Binmore and Herrero (1988a). They produce a model of dynamic matching that also parameterizes search costs and explore the sensitivity of equilibrium to information costs.

8 If the number of firms does not equal the number of households, we may relate the search rate of labourers to the rate each firm's wage offer is solicited by the ratio of labourers to firms. Such a complication would not affect the results if households separate the actions of labourer and firm.

an arbitrary number $i \in [0, \infty)$ offers. The resulting probability $p_0(\mu)$ of receiving no satisfactory job offers then yields the unemployment rate $U(\mu)$.

III. THE NASH DISTRIBUTION IN OFFERED WAGE STRATEGIES

In this section, we characterize the distribution of offered wages and determine the average market wage given the level of search chosen by labourers. A household acting as a firm offers wages according to a mixed strategy as determined by firms' symmetric offered wage distribution. The firm takes as given the (common) reservation wage α^* and average search rate μ of each labourer, the return to capital r, and the exogenous productivity of capital y^*. Its expected total output each period is then given by $y^*d(\alpha)$ where $d(\alpha)$ equals the probability of filling a job offering a wage α. The profit function is then

$$\pi(\alpha) = d(\alpha)(y^* - \alpha), \qquad d(\alpha) \leqq 1.$$

Given the parameters y^* and α^*, we may characterize a symmetric wage-offer distribution as follows:

1. At the beginning of each period, labourers sequentially enter the labour market and instantaneously solicit wage offers for jobs that last one period.[9] A labourer accepts the job in their sample pool offering the highest wage. Labourers receive offers as a poisson process at an average rate μ.
2. Firms offer wages according to a mixed strategy determined by the symmetric distribution of offered wages. This reverse cumulative wage distribution $G(\alpha)$ is defined such that $G(\alpha : \alpha > \alpha_h) = 0$, $G(\alpha : \alpha < \alpha_1) = 1$ where α_h (α_1) is the highest (lowest) offered wage.
3. Given $G(\alpha)$ and $d(\alpha)$, a firm cannot increase its profits by changing its offered wage strategy to any other on the support of $G(\alpha)$.

We first characterize the Nash distribution of wage offers. While no firm will offer a wage less than labourers' common reservation wage α^*, the probability that a firm offers the reservation wage α^* depends on the probability that it will secure a willing labourer and thus depends on the proportion of labourers who receive only a single job offer. We begin by showing that a non-trivial symmetric wage distribution exists and prove that labourers' reservation wage α^* must be an element of the wage distribution. The maximum feasible wage α_h in a stochastic equilibrium only asymptotically approaches the marginal product of labour y^* as information becomes perfect and market frictions become negligible. The support of the wage distribution must then be contained on the domain of feasible wages (α^*, y^*).

9 This assumption of sequential entry of labourers avoids the problem of ties in which the same job is offered to two labourers. A period of unit length is a simplifying assumption only.

PROPOSITION 1. *There does not exist a mass point in the symmetric wage distribution except perhaps at labourers' reservation wage α^*.*

Proof. By contradiction. Suppose that the symmetric wage distribution supports a mass point at a wage $\alpha_0 > \alpha^*$. Because there is a positive probability that a labourer will receive more than one offer, a labourer who receives an offer at a wage α_0 could also receive another offer at a wage α_0. A firm can then offer a wage infinitesimally higher than α_0 and receive a discrete increase in labour supplied and net output for an infinitesimal increase in wage costs, thereby breaking the supposed equilibrium. Consider the lowest market wage α_1. Because there cannot exist a mass point at any wage α exceeding the reservation wage α^*, those firms offering jobs at a wage α_1 secure labourers receiving no other wage offers. Those firms could then lower their offered wage to the reservation wage α^* without any loss in the supply of labourers but with a discrete increase in profits. To complete the proof, stochastic matching implies that no firm can offer a wage y^* and earn a positive return to its capital, since it can never immediately secure a labourer with certainty (probability of one). QED

The competitive model assumes all agents are perfectly informed and predicts a single wage at the marginal product of labour y^*. On the other hand, the model developed by Salop and Stiglitz (1977) predicts only the monopoly price, since each firm acts as a discrimating monopolist. We present an intermediate case in which there exists ex post differential information. We shall show that neither the monopsony wage α^* nor the competitive wage y^* exists as the single wage in this general equilibrium. Hence mass points cannot exist in equilibrium. We next derive the probability that a given job at a wage α will be chosen in a given period.[10]

PROPOSITION 2. *Let $G(\alpha)$ be the symmetric reverse cumulative distribution of offered wages in the market. Then the probability $d(\alpha)$ of filling an available job offering a wage α in a given period is*

$$d(\alpha) = \sum_{i=1}^{\infty} ip_i(\mu)(1 - G(\alpha))^{i-1}, \qquad \text{for all } \alpha \in [\alpha_1, \alpha_h]. \qquad (1)$$

Proof. The probability (α) that a labourer will accept a job offering a wage not less α is given by

$$1(\alpha) = \sum_{i=1}^{\infty} p_i(\mu)(1 - (1 - G(\alpha))^i) \qquad \text{for all } \alpha \in [\alpha_1, \alpha_h], \qquad (2)$$

Where $(1 - G(\alpha))^i$ is the probability that none of i offers exceeds α. By differentiating this expression with respect to the wage α and dividing by the share of

10 For risk-neutral agents, the probability of filling an available job is independent of the duration the job has been available.

available jobs $|G'(\alpha)|$ at a particular wage α, we find the probability that a firm offering a wage α will fill the job in a given period reduces to that stated in the proposition. QED

Given the distribution of demand for jobs offering a wage α, we next determine the symmetric offered wage distribution by equating expected net profits to the equilibrium return to capital once all profitable offered wage niches are filled.

PROPOSITION 3. *A unique non-degenerate symmetric distribution of offered wages $G(\alpha)$ is given by*

$$G(\alpha) = \frac{1}{\mu} \ln \frac{\mu(y^* - \alpha)}{r}. \tag{3}$$

Proof. The density of wage offers requires that net profits $y^* - \alpha$ employed with probability $d(\alpha)$ (from (1)) yields the common equilibrium return r:

$$\sum_{i=1}^{\infty} ip_i(\mu)(1 - G(\alpha))^{i-1}(y^* - \alpha) = r. \tag{4}$$

Dividing both sides of (4) by $(y^* - \alpha)$ gives

$$\sum_{i=1}^{\infty} ip_i(\mu)(1 - G(\alpha))^{i-1} = r/(y^* - \alpha). \tag{5}$$

Solving the left-hand side of (5) gives

$$\mu e^{-\mu G(\alpha)} = \frac{r}{y^* - \alpha}. \tag{6}$$

Differentiating the right-hand side of (4), equating to (5), and simplifying gives

$$1/(y^* - \alpha) = -\mu G_\alpha(\alpha). \tag{7}$$

Solving (6) for $G(\alpha)$ yields

$$G(\alpha) = \frac{1}{\mu} \ln \frac{\mu(y^* - \alpha)}{r}. \tag{7a}$$

QED

This symmetric wage distribution assumes that there exist no offered wage niches yielding a higher than equilibrium return to capital. There are an equal number of unemployed labourers and unemployed units of capital, implying that the equilibrium natural unemployment rate $U(\mu)$ also equals the equilibrium excess capacity rate. The average capacity utilization rate and employment rate is then given by $E(\mu) = 1 - U(\mu) = 1 - p_0(\mu)$ where $p_0(\mu)$ is the proportion of those labourers that do not find jobs. Proposition 3 determines the symmetric distribution of wages, while proposition 1 shows that the lowest market wage is equal to the reservation wage α^*. The highest market wage α_h is determined by setting $G(\alpha_h)$ equal to zero and solving

$$\alpha_h = y^* - \frac{r}{\mu}, \tag{8}$$

which asymtotically approaches the competitive wage y^* as information becomes perfect.

We can now determine the average wage w for employed capital. The average equilibrium return to capital r is the product of the probability $E(\mu)$ of employing a unit of capital and the return to employed capital; that is, $E(\mu)(y^* - W) = (1 - p_0(\mu))(y^* - w)$. The return for firms offering a wage α^* in the symmetric equilibrium is $p_1(\mu)(y^* - \alpha^*)$. Define $q_1(\mu)$ to be the share $p_1(\mu)/(1 - p_0(\mu)) = \mu p_0(\mu)/(1 - p_0(\mu))$ of those labourers who sample a single wage offer out of all labourers who receive at least one offer. We may then determine firms' reaction function that gives their average wage w_F as a function of labourers' search intensity μ and the exogenous marginal product of labour y^* and reservation wage α^*:

$$w_F = y^* - q_1(\mu)(y^* - \alpha^*). \tag{9}$$

This wage locus is upward sloping and concave in labourers' sample rate μ and depends on the effect of search on $q_1(\mu)$:

$$\text{sgn}\,(dw_F(\mu)/d\mu) = -\text{sgn}\,(dq_1(\mu)/d\mu)$$

$$\text{sgn}\,(d^2 w_F(\mu)/d\mu^2) = -\text{sgn}\,(d^2 q_1(\mu)/d\mu^2).$$

Application of L'Hospital's rule to the expression $q_1(\mu) = \mu \exp(-\mu)/(1 - \exp(-\mu))$ evaluated to $\mu = 0$ shows

$$q_1|_{\mu=0} = (\mu/(\exp(\mu) - 1))|_{\mu=0} = (1/\exp(\mu))|_{\mu=0} = 1.$$

Differentiating $q_1(\mu)$ with respect to μ gives[11]

$$dq_1(\mu)/d\mu = (1/(\exp(\mu) - 1))(\exp(\mu) - 1 - \mu \exp(\mu))$$

$$\text{sgn}(dq_1(\mu)/d\mu) = \text{sgn}(\exp(\mu) - 1 - \mu \exp(\mu)) \leqq 0.$$

11 The second derivative shows that $q_1(\mu)$ decreases at an increasing rate:

$$d^2 q_1(\mu)/d\mu^2 = (\exp(\mu)/(\exp(\mu) - 1)^2)(-(2 + \mu) + 2\mu/(\exp(\mu) - 1))$$
$$= (\exp(\mu)/(\exp(\mu) - 1)^3)(-(2 + \mu)(\exp(\mu) - 1) + 2\mu \exp(\mu))$$
$$= (\exp(\mu)/(\exp(\mu) - 1)^3)((\mu - 2)\exp(\mu) + (\mu + 2)) \geqq 0.$$

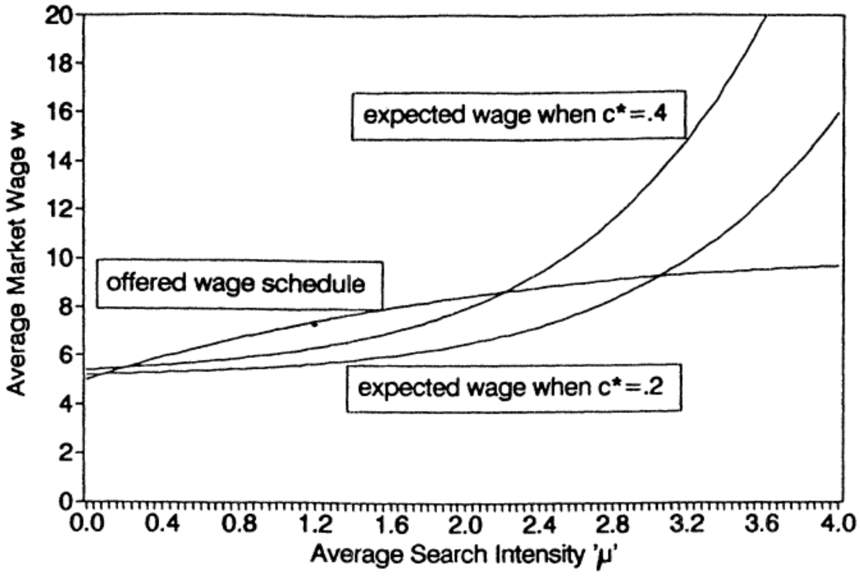

FIGURE 1 Search and the equilibrium wage (for $y^* = 10$ and $\alpha^* = 5$)

Firms' average offered wage schedule $w_F(\mu)$ begins at labourers' reservation wage α^* for a search rate μ that approaches zero and converges to the perfect information competitive wage y^* only as information becomes perfect (i.e., $\mu \rightarrow \infty$). This schedule is increasing in the search rate μ at a decreasing rate, as shown in figure 1.

IV. THE LABOURER'S DETERMINATION OF OPTIMAL SEARCH INTENSITY

We next determine the optimal symmetric level μ of labourers' search given their expected wage w_L if they find a job and their value of leisure (reservation wage) α^* if they do not. We assume that identical risk-neutral, wage-taking labourers have a constant marginal utility of income and a zero discount rate. These households then choose their optimal search intensity to maximize expected income Y^e, where this expected income incorporates households' value of leisure α^* and their expected return to capital r:

$$\max_{\mu} \; Y^e = \max_{\mu} \; (r+E(\mu)w_L+U(\mu)\alpha^*-c^*\mu) = \max_{\mu} \; (r+w_L-p_0(\mu)(w_L-\alpha^*)-c^*\mu),$$

where $c^*\mu$ is the precommitted cost of job sampling sufficient to secure an average number μ job offers and w_L is their expected wage. Noting that $dp_0(\mu)/d\mu$ equals $-p_0(\mu)$, the first-order and second-order conditions with respect to the search intensity are

$$dY^e/d\mu = p_0(\mu)(w_L - \alpha^*) - c^* = 0 \qquad (10)$$

$$d^2Y^e/d\mu^2 = -p_0(\mu)(w_L - \alpha^*) \leqq 0. \qquad (11)$$

The second-order condition for a maximum is globally satisfied because labourers' expected wage w_L cannot be less than their reservation wage α^*. This fact ensures that an interior solution to the first-order condition must be a unique maximum. Inspection of (10) shows that labourers will invest in search only if their expected wage w_L exceeds their value of leisure α^*. We are thus assured of an interior solution $\mu \in (0, \infty)$ to the labourer's maximization problem.

Expressing the first-order condition (10) in terms of the expected wage w_L (as a function of the search intensity μ) gives an implicit definition for the search intensity μ:

$$w_L = \alpha^* + c^*/p_0(\mu) = \alpha^* + c^*\exp(\mu). \tag{12}$$

This equation depends only on the exogenous reservation wage α^* and the marginal search cost c^*. Equation (12) represents labourers' reaction function, mapping between the search intensity $\mu \in (0, \infty)$ and the expected wage $w_L(\mu) \in (\alpha^*+c^*, \infty)$. This locus, expressed as $w_L(\mu)$, is upward sloping and convex in the search intensity μ. Figure 1 shows two expected wage loci for different levels of search costs.

V. EXISTENCE OF MARKET EQUILIBRIUM

Firms' average wage locus is a continuous, upward-sloping, concave schedule as a function of labourers' search rate μ and maps the search intensity $\mu \in (0, \infty)$ into a wage $w_F \in (\alpha^*, y^*)$. Labourers' expected wage locus $w_L : (0, \infty) \longrightarrow [c^* + \alpha^*, \infty)$ is a continuous, upward-sloping, convex schedule that determines labourers' optimal search rate μ as a function of their expected wage (see figure 1). We next combine the two reaction functions for the average wage as a function of search intensity to determine the symmetric market equilibrium with stochastic search and decentralized production and consumption. Solving for a common average received and expected wage (from (9) and (12)) gives

$$y^* - q_1(\mu)(y^* - \alpha^*) = c^*/p_0(\mu) + \alpha^*,$$

$$p_0(\mu)(1 - q_1(\mu)) = c^*/(y^* - \alpha^*). \tag{13}$$

Note that the left-hand side of (13) is independent of market parameters. An application of L'Hospital's rule shows that the left-hand side of (13) is non-negative, equal to zero if $\mu = 0$, and asymptotically converges to zero as information becomes perfect ($\mu \longrightarrow \infty$). Figure 2 compares the left-hand and right-hand sides of (13).[12] The necessary condition for the existence of an equilibrium requires that there exists a closed interval of search intensities μ such that firms' average

12 While it is not possible to verify analytically the shape of the left-hand side of (13), the graph of the function shown in figure 2 is single peaked, positively sloped up to a maximum point (where μ equals 0.156, independent of parameter values), and monotonically decreasing thereafter.

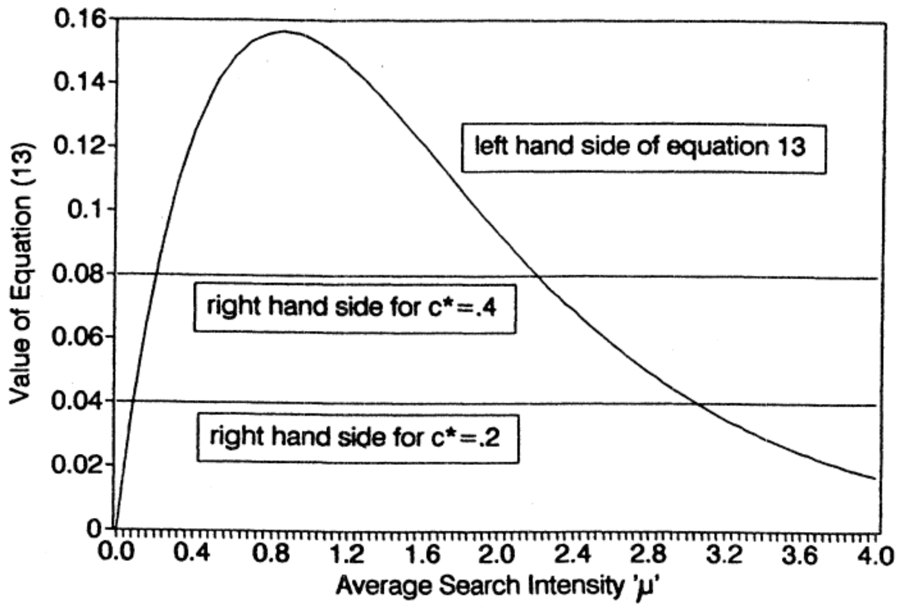

FIGURE 2 Solutions to equation (13) (for $y^* = 10$ and $\alpha^* = 5$)

accepted wage offer $w_F(\mu)$ somewhere exceeds labourers' expected wage $w_L(\mu)$. This requires that the single peaked expression on the left-hand side of (13) must lie somewhere above the horizontal line characterized by the right-hand side of (13). If such a solution exists, there will generally be two roots to (13).[13] The figures compare the intersection points of the left- and right-hand sides of (13) to the intersection of firms' and labourers' wage loci. The lower equilibrium search intensity μ^l represents a Keynesian-type underemployment equilibrium with low wages both causing and caused by labourers' low search rate. The higher search intensity μ^h yields a higher wage and a lower level of frictional unemployment.

VI. SOME COMPARATIVE STATICS RESULTS

A Walrasian auctioneer avoids unrealized gains from trade. However, coordination breakdown is central to Keynes's notion of an underemployment equilibrium in a complex economy with incomplete information. We may now say something about the effects of parameter changes on the decentralized, imperfect information equilibrium. It is convenient to describe the system through the equilibrium average market wage w. We first look at those parameter changes that result in a shift in the firms' reaction function without affecting labourers' expected wage locus. Firms' wage locus shifts upward if a parameter change results in a higher average wage,

13 A sufficient condition for the existence of equilibria is that the combination of exogenous variables on the right-hand side of (13) is less than 0.156: $0.156 \geqq p_0(\mu)(1 - q_1(\mu)) = c^*/(y^* - \alpha^*)$.

holding search intensity constant. This locus shows the following sensitivity to changes in exogenous parameters:

$$w_F(\mu) = (1 - q_1(\mu))Y^* + q_1(\mu)\alpha^*$$

$$dw_F/dy^* = (1 - q_1) > 0$$

$$dw_F/d\alpha^* = q_1 > 0.$$

An increase in the marginal product of labour y^* or the value of leisure α^* unambiguously shifts upward firms' wage locus. An increase in y^* does not affect labourers' reaction function $w_L(\mu)$, however, implying that an increase in y^* increases the equilibrium search level μ^u and average wage at the upper solution but decreases search effort μ^1 and the average wage at the lower solution.

A rise in the reservation wage α^* results in an upward shift in labourers' expected wage schedule w_L as well:

$$dw_L/d\alpha^* = 1 > 0$$

The net result is ambiguous because an increase in the reservation wage α^* induces a rise in both the offered wage and expected wage loci. A rise in marginal search costs c^* has no effect on firms' reaction function but shifts labourers' reaction function upward:

$$dw_L/dc^* = \exp(\mu) > 0.$$

This rise in search costs lowers the equilibrium wage w and the equilibrium search intensity at the upper root and raises the average water and equilibrium search level at the lower root. At this upper root, either a rise in the productivity of labour or a fall in search costs increases the equilibrium search intensity μ^u and induces a fall in unemployment and excess capacity. Consequently, households' welfare rises. The use of output and search incentives as public policy instruments may be unpredictable, however, because the direction of resulting changes is reversed at the lower (Keynesian) root.

1. The social optimum and policy ramifications

A search externality exists because labourers do not acknowledge the effect of their additional search on market competitiveness and the average wage. The decentralized first-order condition for optimal search sets to zero the partial derivative $\delta Y^e/\delta\mu$ of expected income with respect to the search rate, yielding $p_0(\mu)(w - \alpha^*) = c^*$. The optimal level of search maximizes expected income Y^e where output y^* is earned with probability $1 - p_0(\mu)$, and leisure is enjoyed with probability $p_0(\mu)$:

$$Y = (1 - p_0(\mu))y^* + p_0(\mu)\alpha^* - c^*\mu$$
$$= y^* - p_0(\mu)(y^* - \alpha^*) - c^*\mu.$$

The socially optimal, welfare-maximizing level of search then satisfies the first-order condition $dY^e(\mu))/d\mu = 0$, implying that

$$p_0(\mu) = c^*/(y^* - \alpha^*).$$

Households underinvest in search vis-à-vis the welfare-maximizing, pareto optimal solution because they do not acknowledge the effects of their search on the degree of wage dispersion in the market. Labourers set $p_0(\mu)$ to $c^*/(w-\alpha^*)$, implying that an optimal search subsidy of an amount $c^*(w-\alpha^*)/(y^*-\alpha^*) < c^*$ would counteract this externality and increase output by more than the increase in resources devoted to the subsidy.

The mechanisms that result in unambiguous effects on reduced unemployment and higher average wages at the high-wage, low-unemployment solution include increased value of the marginal product of labour and decreased marginal search costs. Both output and search cost subsidies raise the equilibrium search rate μ^u and reduce the natural unemployment rate $U(\mu^u) = p_0(\mu^u)$. In this model, such incentives may then be used to reduce the resulting frictional unemployment rate.

2. Convergence to equilibrium

A dynamic extension of this model would require the specification of adjustment costs for firms and households.[14] Without modelling these costs, we can indicate the direction but perhaps not the speed of adjustment from a state that differs from a static solution.

Consider first the upper root μ^u yielding higher wages, employment, and search intensity. A small increase in the search intensity from the upper solution μ^u increases labourers' expected wage above firms' average wage, resulting in decreased search and a movement back toward equilibrium. A corresponding decrease in the search intensity from μ^u results in labourers' expected wage that is lower than firms' average wage, causing an increase in search until the equilibrium intensity μ^u is again reached. This stable equilibrium yields wage dispersion reminiscent of the Classical, full-employment equilibrium but with some frictional unemployment. For the lower solution μ^1, an increase in the average market wage induces labourers to search more intensively, further increasing wages until the high-wage, high-employment wage is reached. If a perturbation causes an average wage lower than $w(\mu^1)$, however, labourers reduce their search level, resulting in further decreases in average market wages and causing the economy to move towards the degenerate trivial equilibrium with no search or output. For the market to converge on the upper equilibrium, the initial average wage must exceed $w(\mu^1)$. Policy may then be pursued to provide small local changes in the equilibrium only after global changes ensure that the market is directed towards the stable equilibrium.

Interestingly, as marginal search costs c^* go to zero, the model solves for an average market wage w of either the reservation wage α^* or the competitive wage

14 Howitt and McAfee (1988) suggest that adjustment costs may stabilize a dynamic extension of a model such as that presented here.

y^*. Any small perturbations in the lower equilibrium point $w(\mu^1) = \alpha^*$ will move the economy to the full employment solution characterized by the natural (frictional) unemployment as described by Friedman (1968). As a consequence, a trivial or a Keynesian underemployment equilibrium cannot be maintained if search costs are insignificant.

VII. CONCLUSION

We described the general equilibrium in a simple static macroeconomy with imperfect information and stochastic matching. We characterized equilibrium excess capacity, wage dispersion, and the natural unemployment rate and found that a simple economy with costly, non-sequential search may not be characterized by the law of a single wage.

This paper parameterized marginal search (information) costs. In the limit as these information costs go to zero, a stable, competitive solution is obtained, characterized by full employment and a market wage equal to the value of labourers' marginal product. However, the market is generally characterized by a suboptimally high frictional (natural) unemployment rate and low average wage as a consequence of non-cooperative behaviour. Indeed, there exist two solutions to the problem, one of which exhibits coordination problems characteristic of a Keynesian underemployment equilibrium. In other words, full employment consistent with a Walrasian auctioneer (as search costs go to zero), frictional unemployment, and Keynesian underemployment arise within a single general equilibrium model of the macroeconomy.

Public policy may be used to ensure that the economy is at the higher of two non-trivial equilibrium wages. Once the economy is directed to this high-wage solution, fine tuning of average wages and unemployment may be appropriate. Otherwise, seemingly counter-intuitive effects of employment policy instruments can result. Finally, search and wage externalities imply firms individually find it unprofitable to offer higher wages even though increased wages are collectively welfare maximizing. Consequently, search subsidies or other such instruments may be used to counteract these externalities and obtain the utility maximizing, pareto optimal level of output.

APPENDIX: NOTATIONAL GLOSSARY:

α^* = reservation wage (or value of leisure)
c^* = marginal and average search costs
$d(\alpha)$ = the probability of filling an available job at a wage α
$E(\mu)$ = the market employment rate
$G(\alpha)$ = reverse cumulative distribution of firms by offered wage
i = dummy index variable for the number of offers received
$1(\alpha)$ = proportion of labourers finding jobs of wage greater than α
p_i = proportion of labourers receiving i offers (exogenous to firms)

q_i $= p_i(\mu)/(1 - p_0(\mu))$
r = rental cost of capital
μ = average arrival rate of job offers (exogenous to firms)
μ^l = lower equilibrium search rate
μ^u = upper equilibrium search rate
$U(\mu)$ = market unemployment rate
w = average equilibrium wage
w_F = firms' average wage
w_L = labourers' expected wage if employed
y^* = value of the marginal product of labour
Y^e = households' expected income

*Denotes exogenous variables

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