# Tax Policy and the Supply of Exhaustible Resources: Theory and Practice

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## I. INTRODUCTION

Taxes, subsidies, and price controls are ubiquitous features of extractive industries. They appear at every stage of production and consumption. They take many forms, including severance taxes, royalties, depletion allowances, and profits taxes. And their size is not small. For example, depletion allowances have been as high as 33⅓% of total revenues and price controls have kept domestic prices at less than half world prices.

There are many reasons why efficiency and equity might require governments to intervene in private markets for exhaustible resources. For example, extraction may proceed too fast if there are common-pool problems in exploration or extraction, and it may proceed too slowly if markets are monopolized. In addition, unregulated extraction patterns may not be optimal if social and private discount rates differ or if extraction creates an externality.

The design of optimal taxes is never a simple matter. Because extraction is inherently intertemporal, however, it is more complicated for mineral commodities. The difficulty of designing tax policy that achieves a particular objective is illustrated by the fact that a tax levied in any time period can have repercussions in all future time periods and, if it is anticipated, it can affect past extraction.

Many people have studied the theory of taxation of exhaustible resources, including Burness (1976), Sweeney (1977), Dasgupta, Heal, and Stiglitz (1980), and Conrad and Hool (1981). These authors simplify matters by assuming that ore is sold directly without prior processing, the quantity of ore extracted is the sole factor that determines extraction cost, and taxes and subsidies apply directly to the unprocessed ore.[1]

In contrast to the theoretical literature on resource taxation, many empirical studies such as those by the U.S. General Accounting Office (1981) and Foley and Clark (1982) are based on static models of profit maximization. No attempt is made to capture the intertemporal effects of taxation on the supply of exhaustible resources.

This paper continues the analysis of Lewis and Slade (forthcoming). It is assumed here that processing takes place after extraction and prior to sale, inputs other than ore enter the extraction and processing cost functions, taxes and subsidies can be applied at any stage of production, and tax effects are intertemporal (based on dynamic profit-maximizing calculations).

The paper differs from Lewis and Slade (and most of the earlier literature on resource taxation) in allowing for less restrictive extraction and processing technologies, depletion effects (costs shifting up with cumulative extraction), less than complete recovery of the ore body, multiple grades, and multiple outputs (byproducts and coproducts).[2]

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[1] Conrad and Hool (1981) are the sole exception in the literature. They compare taxes on final products to taxes on ore. In their model, however, ore is the only input and metal output is a constant fraction of ore input.

[2] The general model allows for multiple outputs and multiple grades. Due to data limitations, however, the empirical implementation deals with a single output, copper, and a uniform grade of ore.

The difficulty of extending the theoretical results on resource taxation to handle the above complications has led to the use of numerical techniques. Numerical techniques have the additional advantage of allowing for assessment of magnitudes as well as directions of tax effects.

A general model for assessing the effects of taxation on resource extraction is developed. The model is then econometrically estimated for a particular copper-producing firm; that is, the firm's variable-profit function is obtained. Numerical nonlinear optimization techniques are used to solve for optimal extraction and processing paths for the no-tax case. Various sorts of taxes and subsidies are then imposed on the profit function and new optimal extraction and processing paths are computed. Finally, the undistorted (no-tax) and distorted paths are compared to determine the direction and magnitude of tax effects. It is found that the change in extraction paths resulting from a particular tax depends on where it falls, on how it affects input and output prices, and on the ease of substituting between the resource and other inputs in processing.

## II. THEORETICAL MODEL

In this section, the model for assessing the effects of tax policy on the production decisions of a firm is developed. Prices are assumed to be exogenous to the firm and to be unaffected by taxes and subsidies.[3] Firms have perfect information about prices in all time periods. Finally, the firm has a finite planning horizon of length $T$, where $0<T<\infty$.

It is assumed that the firm has production possibilities that are described by a single-period technology set, $\mathrm{T}=\{Q,X,S_0,S_1\}$, where

- $Q$ is a vector of "metal" outputs, $Q=(Q_1,Q_2, \ldots,Q_k)$;
- $X$ is a vector of variable-inputs, $X=(X_1,X_2, \ldots,X_n)$;
- $S_0$ is a vector of beginning-of-period stocks of "ore"; $S_0=(S_0^1,S_0^2, \ldots,S_0^m)$, where each stock corresponds to a different grade; and
- $S_1$ is a vector of end-of-period stocks of "ore," $S_1 = (S_1^1,S_1^2, \ldots,S_1^m)$, which become the initial stocks in the next period.

The words "metal" and "ore" are used to denote processed and unprocessed resources, respectively. "Ore" could be crude petroleum, in which case "metal" might be gasoline.

The technology set T describes the tradeoffs that exist between producing more current outputs, $Q$, using fewer current inputs, $X$, and running down the end-of-period stocks, $S_1$. These tradeoffs are conditional on the beginning-of-period stocks, $S_0$.

Dual to the technology set is the firm's variable-profit function, $\Pi(Q,P,V,S_0,S_1)$, where

- $P$ is a vector of metal prices, $P=(P_1,P_2, \ldots,P_k)$ and
- $V$ is a vector of variable-input prices, $V=(V_1,V_2, \ldots,V_n)$.

The firm's profit-maximizing decision is thus to choose vectors of extraction rates $R_t=S_{t-1}-S_t$ (a grade-selection profile) and output rates $Q_t$ in each period in such a way as to maximize its discounted profit stream. That is, the firm wishes to

$$\max_{R_t,Q_t} \sum_{t=1}^{T} \delta^{t-1}\Pi(Q_t,P_t,V_t,S_{t-1},S_t) \qquad [1]$$

subject to $R_t = S_{t-1} - S_t$

and

$$\sum_{t=1}^{T} R_t \leqslant \bar{S},$$

where $\delta = 1/(1+r)>0$ is the constant discount factor ($r$ is the constant discount rate) and $\bar{S}$ is the vector of stocks of ore in the ground when the mine opens. In what follows, equation [1]

[3] Most mineral commodities are sold in world markets. Taxes, however, are often levied by smaller jurisdictions such as a state or province. A tax imposed locally should therefore not affect the price at which a firm can sell its output.

is used to assess tax effects for a particular mining firm.

## III. THE WHITE PINE MINE

Implementation of the model described in the last section requires data on costs and revenues for a specific extractive firm. The ideal data source for this purpose is company annual reports. Many mining companies, however, are large multinational conglomerates producing multiple outputs in diverse regions of the world. If a company is highly diversified, reported revenue and cost figures have little economic meaning. There are, however, a few smaller mining firms that are highly specialized and operate only one mine. Copper Range is one such firm and for this reason was chosen for empirical implementation.[4]

Copper Range Company is a copper-mining firm that owns and operates only one mine, the White Pine mine on the Upper Peninsula of Michigan. The mine opened in 1955 and continues to operate today. Copper Range, however, was acquired by the Louisiana Land and Exploration Company in 1977. Data pertaining to years after 1976 are therefore useless for the purpose of this study.

The data used here consist of the initial stock of ore (the stock when the mine opened), the quantity of ore extracted each year, the quantity of metal produced each year,[5] yearly copper prices, and the prices and quantities of variable inputs. These data are available from the author upon request.

The White Pine mine is a large underground facility that extracts from a strata-bound deposit with sulfide ores. The mine is a comparatively high-cost operation utilizing a mining technique known as "room and pillar." This system of mining expands the perimeter of mined-out areas year by year, increasing the distances that men, materials, supplies, and ore must be transported. Mining costs thus rise with cumulative production.

Mining at White Pine is conducted around a perimeter embracing an area of about seven square miles. Around this perimeter there are approximately 20 mining sites containing ore of various grades. As the grade varies, so also do mining conditions and costs. The concentrator, however, is designed to accept only a narrow range of grades. That is, the concentrator operates at lowest cost when grade is approximately 1%. Because costs increase sharply when much higher or lower grades are processed, the operators of the mine combine production from the various areas to supply ore to the concentrator that is fairly uniform in grade. Very little grade variation therefore appears in the data.[6] Table 1 shows the average grade of ores sent to the mill in the years 1972–1975.

Even though the grade of ore sent to the mill is fairly constant, the amount of metal recovered from the ore varies, depending on economic conditions. If the price of copper is high relative to the cost of variable inputs, it pays the company to process the ores more intensively. The second line of Table 1 shows the average yield of ores sent to the mill in the same years, where yield is defined as metal produced divided by ore mined. Unlike grade (the quantity of metal in ore in the ground), which is a geologic fact, yield depends on the technology available and on production decisions. Yield is thus a measure of recovery efficiency and is bounded from above by grade (that is, it is impossible to recover more metal than the ore contains). When yield equals grade, recovery efficiency is 100%.

Table 1 shows that yield varied more than grade over the 1972–1975 period. It therefore seems reasonable to assume that changes in

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[4] While the world copper industry once was a tight oligopoly, in recent years it has become much less concentrated and is now generally considered to be workably competitive. For example, Foley and Clark (1982) model it as a competitive industry. Copper Range is one of the smaller firms in the industry and thus the price-taking assumption should be realistic.

[5] Silver is produced as a byproduct of copper mining. The equipment to recover silver from the ore was not installed, however, until 1969. The period of the data is thus too short (eight observations) to obtain meaningful econometric estimates for the multiple-output case.

[6] In 1976, the average grade of ore mined was 1.4%. Cost per ton of ore that year was much higher and this information is used to fit the spliced function $\Theta$ described in section IV*c* (to choose the parameter $\mu$). Unfortunately, there is not enough grade variation in the data to estimate the grade-cost relationship statistically. The year 1976 was not used in the estimation.

TABLE 1

AVERAGE GRADE AND YIELD OF ORES MINED (%)

| | 1972 | 1973 | 1974 | 1975 |
|---|---|---|---|---|
| Average Grade of Ore Sent to the Mill | 1.010 | 1.000 | 1.050 | 1.010 |
| Average Yield of Ore = Metal/Ore | .86 | .88 | .81 | .79 |
| Recovery Efficiency = Yield/Grade | 85 | 88 | 77 | 78 |

*Source:* Copper Range Co., Annual Report, 1976.

the metal-to-ore ratio are due principally to more or less intensive processing of the ores rather than to changes in the grade of ores mined.

The information about the White Pine mine contained in this section is used in specifying the firm's variable-profit function that is estimated in the next section.

## IV. THE ECONOMETRIC ESTIMATES

### IV*a. Functional Form*

In order to estimate the firm's variable-profit function, it is necessary to specify its functional form. There are a number of flexible forms for profit functions (see Diewert 1974, for example). For considerations of data availability and degrees of freedom, however, the profit function used here is restricted somewhat.

The production of most metals and fuels consists of two stages—extracting the ore from the ground (mining) and refining or processing it in a mill, smelter, or refinery. A variable-cost function should therefore have two parts, one for extraction and another for processing. Data on input use, however, are not generally separated by stage of production. I therefore specify a multiplicatively separable variable-cost function of the form

$$VC(V,R,S_0,Y) = c(V)f(R,S_0)h(Y) \qquad [2]$$

where

$VC$ is total variable cost;
$V$ is a vector of variable-input prices;
$R$ is the rate at which ore is extracted;
$S_0$ is the beginning-of-period stock of ore; and
$Y$ is average yield (= metal produced/ore mined).

The function $f$ pertains to the mining stage of the operation whereas $h$ pertains to processing. Variable-inputs are used in both mining and processing stages. Variable costs are expected to increase with the rate of extraction, with the degree of processing, and with cumulative extraction ($VC_R>0$, $VC_Y>0$, and $VC_{S_0}<0$, where a subscripted function denotes the partial derivative of the function with respect to the variable of the subscript).

The variable-profit function is then

$$\Pi(P,V,R,S_0,Y) = P \cdot R \cdot Y - VC(V,R,S_0,Y) \qquad [3]$$

where $P$ is metal price.

There are two variable inputs—labor and a residual input that consists of energy and raw materials. A translog functional form was chosen for $c$ and $f$ (Christensen, Jorgenson, and Lau 1971).[7] $h'$ and $h''$ are expected to be positive. An exponential form (which satisfies these conditions) was chosen for $h$.

The variable-cost function to be estimated is thus

$$\begin{aligned} \ln(VC) = \alpha_0 + {}_1\ln(V_1/V_2) + \ln(V_2) \\ + 1/2\gamma(\ln(V_1/V_2))^2 + \beta\ln(R/S_0) \\ + \ln(S_0) + 1/2\psi(\ln(R/S_0))^2 + \eta Y. \end{aligned} \qquad [4]$$

Using Shephard's (1953) lemma, cost-share equations for each variable input are obtained from the variable-cost function.

$$V_i \cdot X_i / VC = \alpha_i + \gamma\ln(V_i/V_j) \quad i=1,2 \quad \alpha_2 = 1-\alpha_1 \qquad [5]$$
$j = 1(2)$ when $i = 2(1)$.

[7]When the standard restrictions of symmetry and linear homogeneity in prices are applied, the translog with two arguments reduces to a particularly simple functional form which is quadratic in the logarithm of the price ratio.

### IV*b*. *Stochastic Specification and Estimation Technique*

It is assumed that [4] is an exact representation of the variable-cost function and that any deviations from cost minimization are due to random errors. The latter are modeled by appending additive disturbance terms to each of the equations in [4] and [5].

Because the cost shares in [5] sum to one, the sum of the disturbance terms is zero at each observation, implying that the error covariance matrix is singular and nondiagonal. The second cost-share equation was therefore dropped. A maximum-likelihood-estimation procedure was used to estimate the variable-cost function and the remaining cost-share equation as a seemingly-unrelated-regressions system.[8] This procedure has the advantage that it is invariant to the cost-share equation deleted.

### IV*c*. *Empirical Results*

Table 2 shows the parameter estimates and their corresponding asymptotic *t*-statistics. The coefficient $\gamma$ is not shown because when the variable-cost function was estimated with the quadratic price-ratio term, the estimated cost function was not concave in prices. The estimated function with $\gamma$ constrained to equal zero is concave. In addition, $VC_R>0, VC_Y>0$ everywhere, and $VC_{S_0}<0$ whenever $R>.035S_0$, a condition always met in the simulations reported in the next section. Also, $VC_{R^2}>0$, $VC_{Y^2}>0$, and $VC_{S_0R}<0$ (the marginal-cost curve shifts up as the stock of ore is depleted). The variable-profit function therefore satisfies all desired regularity conditions and is thus well-suited for numerical optimization (declared optima will be global maxima).[9]

The function $h$ is a multiplicative factor that shows how costs increase as the ore is refined more intensely. As noted in the last section, there is a theoretical maximum yield (it is impossible to extract more metal than the ore contains). As long as $Y$ is in the range of its observed values, the estimated cost function $\hat{h}$ is an accurate representation of the

TABLE 2
PARAMETER ESTIMATES

| | Estimate | Asymptotic t-statistic |
|---|---|---|
| $\alpha_0$ | 3.470 | 3.7 |
| $\alpha_1$ | .399 | 32.5 |
| $\beta$ | 2.576 | 4.1 |
| $\psi$ | .469 | 2.5 |
| $\eta$ | .321 | 1.7 |
| Log-Likelihood Function = 51.9 | | |

cost-yield relationship. Costs, however, increase very rapidly as the theoretical maximum yield is approached. The estimated function $\hat{h}$ underestimates costs for yields larger than those observed.

To get around this problem, another function $\Theta$ was spliced into $\hat{h}$ at the maximum observed yield. $\Theta$ was chosen to satisfy

$$\Theta(Y,\mu)=\hat{h}(Y) \quad [6]$$
$$\Theta_Y(Y,\mu)=\hat{h}'(Y) \quad [7]$$

and

$$\Theta_{Y^2}(Y,\mu)=\mu\hat{h}''(Y) \quad [8]$$

at the point of the splice (where $\mu$ is a real number greater than one). The spliced function is thus continuous with continuous first derivatives. However, the second derivative of $\Theta$ is $\mu$ times as large as the second derivative of $\hat{h}$ at the point where the two functions join. $\mu$ is thus a parameter that determines how rapidly costs increase as $Y$ approaches the theoretical maximum.

$\Theta$ was chosen to be a polynomial of the form

$$\Theta(Y,\mu)=\nu_1(\mu)Y+\nu_2(\mu)Y^2+\nu_3(\mu)Y^3. \quad [9]$$

[8] The nonlinear-regression option of SHAZAM (White 1978) was used to estimate the system of equations.

[9] It is important that $\Pi$ be well-behaved over a wide range of values assumed by its arguments because it will be used in simulations where variables take on values outside their historic ranges. There are thus tradeoffs between flexibility and regularity.

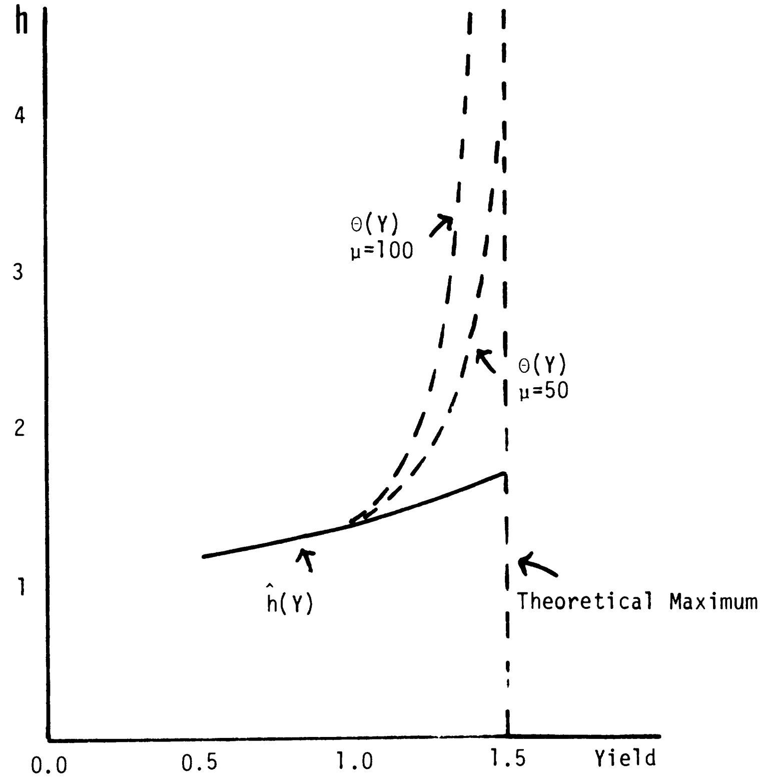


FIGURE 1
The Yield-Cost Relationship for Two Values of μ

Equations [6–8] were used to determine the coefficients $v_1$, $v_2$, and $v_3$ as functions of $\mu$.

The function $h$ that forms part of the variable-profit function is defined by

$$h(Y) = \hat{h}(Y) \qquad Y \leq Y_{max}$$

$$h(Y) = \Theta(Y, \mu) \qquad Y > Y_{max}$$

where $Y_{max}$ is the maximum yield observed in the data used for the estimation.

Figure 1 shows the estimated function $\hat{h}$ and two functions $\Theta$ corresponding to $\mu = 50$ and 100, respectively. A value of $\mu = 100$ was used in the simulations reported in the next section.

## V. NUMERICAL SIMULATIONS

### Va. *General Method*

This section reports results of numerical-simulation exercises using the estimated variable-profit function. The firm's maximization problem is to choose extraction rates and yields in each time period so as to maximize its discounted profit stream. That is, it wishes to

$$\max_{R_t, Y_t} \sum_{t=1}^{T} \delta^{t-1} \Pi(P_t, V_t, R_t, S_{t-1}, Y_t, \Gamma_t) \qquad [10]$$

subject to

$$\sum_{t=1}^{T} R_t \leq \bar{S},$$

where $\Gamma_t$ is a vector of tax variables. Royalties, severance taxes, profits taxes, depletion allowances, and price controls are considered.

A royalty is a tax that is a constant proportion of final-product price. The symbol $\Gamma_1$ is used to denote the constant royalty rate. A severance tax is a constant-dollar tax per unit of ore extracted, denoted by $\Gamma_2$.[10] With a profits-tax, a constant proportion of variable profits is collected; $\Gamma_3$ denotes the profits tax rate. The variable-profit function with these taxes is thus

$$\Pi(P_t, V_t, R_t, S_{t-1}, Y_t, \Gamma_t)$$
$$= (1-\Gamma_3)((1-\Gamma_1)P_t R_t Y_t$$
$$- VC(V_t, R_t, S_{t-1}, Y_t) - \Gamma_2 R_t). \qquad [11]$$

Initially, the extraction and processing patterns that would result in the absence of taxation are characterized; these patterns provide a benchmark. Subsequently, the extent to which specific taxes, subsidies, and controls cause departures from this benchmark is analyzed. Although departures from the tax-free outcome are referred to as distortions or biases, there are, of course, reasons why the tax-free outcome may not be optimal. Some of these are mentioned in the introduction. Nevertheless, it is useful to have a standard to use in measuring the direction and magnitude of changes in extraction and processing patterns that result from different tax-policy instruments.

A constrained nonlinear optimization technique was used to solve the maximization problem [10] for different values of $\Gamma_t$.[11] The three principal effects of taxation that we observe are a tilting of the extraction path, changes in total ore extracted over the period, and changes in total metal produced. Tilting occurs when more (less) ore is extracted in earlier time periods which is offset by lower (higher) extraction rates in later periods. Tilting can occur with or without changes in cumulative ore extraction or changes in total metal production.

For all tax configurations, a discount rate of $r = .05$ and a planning horizon of $T = 16$ years were used. Three relative-price paths were considered. Of interest is the rate at which output price appreciates relative to input prices. For simplicity it is assumed that input prices are constant at their historic means and that

$$P_t = (1+a)P_{t-1} = (1+a)^t P_0, \qquad [12]$$

where $a$ is the constant rate of output-price appreciation and $P_0$ is the historic mean copper price. $a$ takes on the values .00, .04, .08.[12]

Table 3 gives summary statistics for the tax-free outcome. It shows cumulative ore extracted, $\bar{R}$ ($10^6$ tons), cumulative metal produced, $\bar{Q}$ ($10^6$ lbs.), and the average yield of ores mined, $\bar{Y} = \bar{Q}/(20\bar{R})$, for the three values of $a$.

Figure 2 shows extraction and processing paths for the same values. It can be seen that higher rates of output-price appreciation cause cumulative extraction and metal production to increase. In addition, when output price appreciates faster than the rate of interest (when $a > r$), ore appreciates in present value and extraction is thus delayed (the path is tilted towards the future). In contrast, when $a$ is less than $r$, the present value of ore falls over time and the extraction path is tilted towards the present. Finally, when $a$ is approximately equal to $r$, producers are indifferent about when they extract and the path is approximately flat.

To compute the distortions due to tax policy, it is necessary to compare outcomes with

TABLE 3

EXTRACTION AND PROCESSING WITHOUT TAXES

| a | $\bar{R}$ Cum. Ore Extracted ($10^6$ tons) | $\bar{Q}$ Cum. Metal Produced ($10^6$ lbs) | $\bar{Y}$ Average Yield of Ores Mined (%) |
|---|---|---|---|
| .00 | 136 | 2951 | 1.089 |
| .04 | 152 | 3372 | 1.109 |
| .08 | 160 | 3627 | 1.136 |

[10] Some states levy severance taxes on a value basis. Such taxes would be classified as royalties here.

[11] Various optimization routines from the University of British Columbia's NLP (1978) were used to solve this problem.

[12] $a$ is assumed to be positive because in most exhaustible-resource models the undiscounted price of the resource rises over time. Resource prices can fall (see Pindyck 1978 and Slade 1982, for example), but eventually they rise.

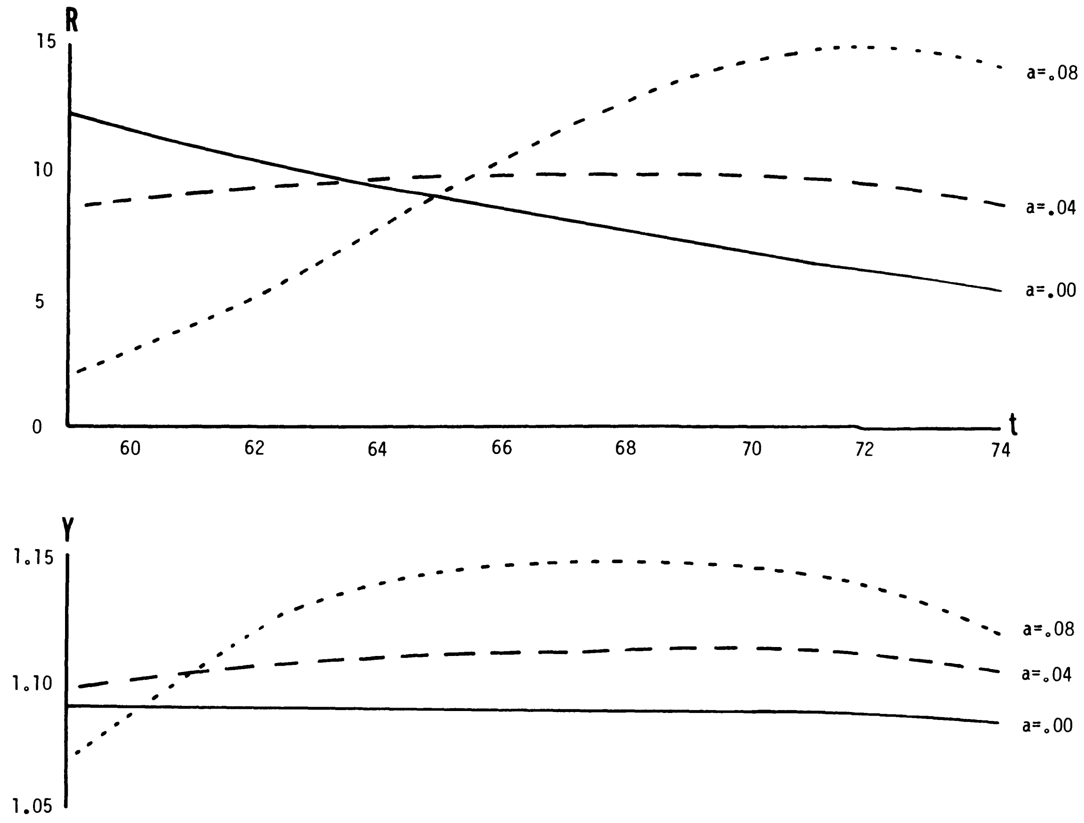


FIGURE 2
TAX-FREE EXTRACTION PATHS

and without taxes for the same value of *a*. When a particular tax or subsidy $\Gamma$ is imposed, the maximization problem [10] is recomputed to obtain $Q_{\Gamma_t}$, the optimal amount of metal production in each period *t* under $\Gamma$. A measure of the distortion due to $\Gamma$ is then

$$D_\Gamma = \sum_{t=1}^{T} \delta^{t-1} | Q_t - Q_{\Gamma_t} | P_t . \qquad [13]$$

$D_\Gamma$ is the present value of the magnitude of the difference between actual and "optimal" extraction paths.[13]

## V*b*. *Royalties*

A royalty, which is a constant fraction $\Gamma_1$ of final-product price, reduces the effective output price in each period. Because the size of the tax varies with output price, its intertemporal effects depend on the path of discounted prices. And because royalties reduce the profitability of extraction and processing, they cause changes in the total amount of ore and metal recovered over the planning period.

It is well known in the theoretical literature that when it is always optimal to extract the entire ore body (when only tilting can occur), the direction of the tilt due to a royalty depends on the relative sizes of *a*, the rate of price appreciation, and *r*, the discount rate. In the simple model where $Q = R$ (no process-

[13] $D_\Gamma$ should not be considered a deadweight or welfare loss. It is difficult to compute a deadweight loss without knowing how the resources released (miners, for example) are employed.

ing occurs), royalties lead to conservation (have no effect on extraction or lead to more rapid depletion) depending on whether $a$ is less than (equal to or greater than) $r$.

Lewis and Slade (forthcoming) show that when the processing-production function is Cobb-Douglas, $Q = R^{\alpha}X^{(1-\alpha)}$, $(1+a)^{1/\alpha}$ must be compared to $(1+r)$.
That is, if

$$(1+a)^{1/\alpha} \begin{array}{l} < \\ = \\ > \end{array} (1+r) \quad \begin{array}{l} \text{royalties lead to conservation} \\ \text{royalties are neutral} \\ \text{royalties lead to rapid depletion.} \end{array} \qquad [14]$$

Because $0<\alpha<1$, royalties are more apt to lead to rapid depletion when processing occurs prior to sale.

In the current, less restrictive model with stock effects in the cost function, less than complete recovery of the ore body, and more general substitution possibilities between $R$ and $X$, it is impossible to derive simple rules that determine the effects of a royalty. Whether the principal effect is a tilting of the extraction path or a lowering of the entire path is not known in general and can only be determined empirically.

Simulations corresponding to royalties of 5% and 10% were run; Table 4 summarizes the results. The upper half of this table shows cumulative ore extracted, $\overline{R}$, cumulative metal produced, $\overline{Q}$, the average yield of ores mined, $\overline{Y}$, and the distortion due to the royalty, $D_{\Gamma}$. The lower half shows percent changes in these variables. Figure 3 compares the tax-free extraction paths to those obtained under a royalty of 10%.

Table 4 shows that a royalty causes both $\overline{R}$ and $\overline{Y}$ to fall so that metal production falls more than ore extraction. This behavior is intuitively plausible because the tax on metal price reduces the shadow price of ore as well as the price of metal and therefore causes less ore to be extracted and less metal to be refined from a given amount of ore. The effects are sizable. For example, when $a = 0$, a 10% royalty causes an 8th% reduction in cumulative metal production valued at $78 million.

Figure 3 shows that the principal effect of royalties is to shift the entire extraction path down; crossing is not generally observed. According to the conventional wisdom, royalties

TABLE 4

THE EFFECTS OF A ROYALTY

| a | $\Gamma_1$ | $\overline{R}$ Cum. Ore Extraction ($10^6$ tons) | $\overline{Q}$ Cum. Metal Production ($10^6$ lbs) | $\overline{Y}$ Average Yield (%) | $D_{\Gamma}$ ($10^6$ \$) |
|---|---|---|---|---|---|
| .00 | .05 | 131 | 2848 | 1.086 | 37.2 |
| .04 | .05 | 149 | 3292 | 1.105 | 34.6 |
| .08 | .05 | 157 | 3565 | 1.132 | 36.6 |
| .00 | .10 | 126 | 2735 | 1.082 | 77.5 |
| .04 | .10 | 145 | 3203 | 1.102 | 72.4 |
| .08 | .10 | 155 | 3497 | 1.128 | 76.3 |
| a | $\Gamma_1$ | %Δ Cum. Ore Extraction | %Δ Cum. Metal Production | %Δ Average Yield | |
| .00 | .05 | −3.7 | −3.9 | −.29 | |
| .04 | .05 | −2.7 | −2.9 | −.34 | |
| .08 | .05 | −1.9 | −2.1 | −.34 | |
| .00 | .10 | −7.4 | −7.8 | −,64 | |
| .04 | .10 | −4.6 | −5.0 | −.61 | |
| .08 | .10 | −3.1 | −3.6 | −.70 | |

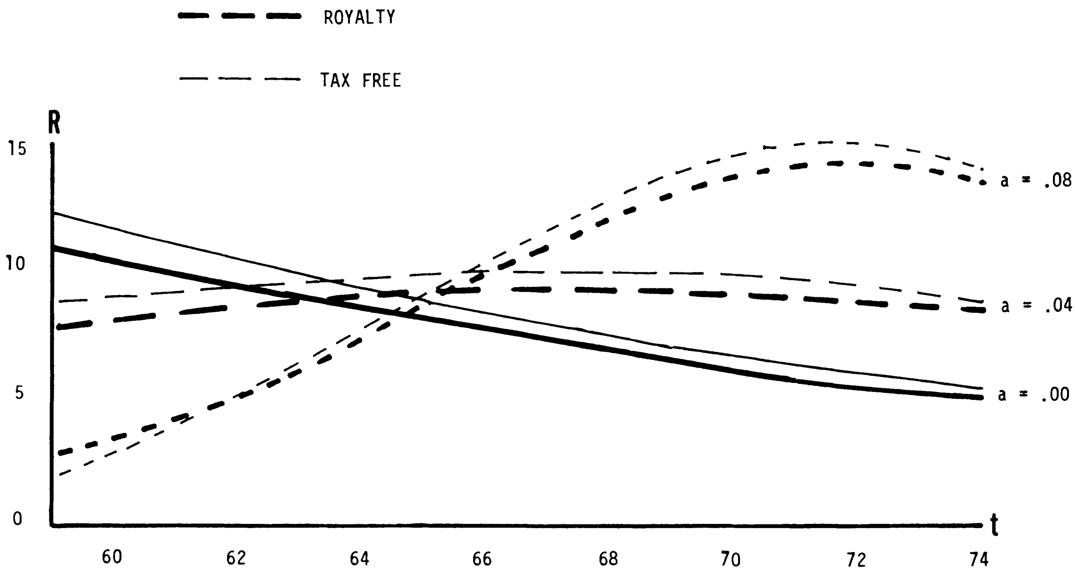


FIGURE 3
EXTRACTION PATHS WITH AND WITHOUT ROYALTIES

conserve the resource.[14] In none of the cases examined, however, can royalties be said to lead to conservation. While it is true that in some cases less ore is consumed in early time periods, lower consumption early on does not result in higher consumption later.[15] Instead, more ore remains in the ground at the end. It is interesting to note that the only tilt observed occurs when $a = .08$. In this case, the royalty results in higher consumption in early periods followed by lower consumption near the end.

### V*c*. *Severance Taxes*

A severance tax, which is a constant-dollar tax $\Gamma_2$ per unit of ore extracted, causes the profitability of extraction to fall. In addition, it causes the cost of ore to increase relative to the prices of other inputs. The imposition of a severance tax is therefore expected to result in a fall in cumulative ore extraction (due to the increase in the cost of ore) accompanied by a rise in the optimal amount of processing (due to the fall in relative prices of the variable inputs). These two effects of a severance tax, which work in opposite directions on metal production, cannot be detected in models where ore is sold directly. In such models, ore and metal production always fall by the same amount when a severance tax is introduced (because $Q = R$).

Table 5 summarizes the effects of severance taxes of $.50 and $1.00 per ton. These values were chosen to result in approximately 5% and 10% changes in variable costs. The results are as predicted. It is optimal to extract less ore and to process it more intensively. The change in metal production is therefore smaller than the change in ore extraction.

### V*d*. *Profits Taxes and Depletion Allowances*

A tax on profit at a constant percent, $\Gamma_3$, is nondistortionary in that it does not affect extraction and processing decisions for an operating mine. However, because profitability is a determinant of exploratory activity as well

[14] By "the conventional wisdom" I mean results obtained from theoretical models with $Q = R$ and $a < r$.

[15] In the theoretical exhaustible-resource literature, a tax is usually considered to encourage conservation if it results in a fall in consumption today followed by increased consumption in later years (that is, if it saves the resource for the future).

as of investment in new facilities (aspects of mining that are beyond the scope of this paper), profit taxes result in lower output and can thus be distortionary in a different sense.

When, however, a profits tax is combined with a percentage depletion allowance, it also distorts extraction decisions for an operating mine. A percentage depletion allowance is a subsidy such that a fixed proportion $\Gamma_4$ of the current value of output is exempt from profit taxes. The effect of a percentage depletion allowance is to raise the apparent price facing the firm from $P$ to $(1+\rho)P$, where $\rho = \Gamma_4\Gamma_3/(1-\Gamma_3)$ (Sweeney 1977). A percentage depletion allowance thus has the effect of a negative royalty equal to $-\rho$.

Simulations were run with a profits tax $\Gamma_3$ equal to 50% combined with percentage depletion allowances $\Gamma_4$ of 5% and 10%. This tax scheme corresponds to royalty rates $\Gamma_1$ of minus 5% and minus 10%. Table 6 summarizes the results (which are similar but opposite in sign to those discussed in section V*b*).

### V*e*. *Price Controls*

A price control is effectively a tax on output equal to the difference between the uncontrolled and controlled prices. The effect of the control thus depends on the behavior of this gap over time. It also depends on whether or not the control was anticipated.[16] To illustrate this point, two sorts of simulation experiments were performed. In both cases, the price control results in constant undiscounted prices after period eight. For the first experiment, however, the imposition of the price control was anticipated whereas for the second the control takes producers by surprise. Table 7 summarizes these results.[17] Unanticipated price controls result in a larger reduction in extraction than do controls that were expected. This happens because more extraction takes place in earlier years when producers are aware that prices will be controlled in the future.

Figure 4 illustrates the pattern of changes in extraction paths. It can be seen that the un-

[16] The distinction between anticipated and unanticipated imposition can also be made for other sorts of taxes.

[17] Table 7 does not show value distortions because it is not clear at what price to value the metal.

TABLE 5
THE EFFECTS OF A SEVERANCE TAX

| a | $\Gamma_2$ ($/ton) | $\bar{R}$ Cum. Ore Extraction ($10^6$ tons) | $\bar{Q}$ Cum. Metal Production ($10^6$ lbs) | $\bar{Y}$ Average Yield (%) | $D_\Gamma$ ($10^6$ $) |
|---|---|---|---|---|---|
| .00 | .5 | 131 | 2855 | 1.091 | 35.1 |
| .04 | .5 | 149 | 3316 | 1.110 | 25.3 |
| .08 | .5 | 158 | 3590 | 1.137 | 13.6 |
| .00 | 1. | 126 | 2748 | 1.092 | 73.5 |
| .04 | 1. | 146 | 3255 | 1.112 | 52.6 |
| .08 | 1. | 156 | 3551 | 1.138 | 28.4 |
| a | $\Gamma_2$ | %Δ Cum. Ore Extraction | %Δ Cum. Metal Production | Average Yield | |
| .00 | .5 | −3.7 | −3.3 | .15 | |
| .04 | .5 | −2.0 | −1.7 | .11 | |
| .08 | .5 | −1.3 | −1.0 | .10 | |
| .00 | 1. | −7.4 | −6.7 | .29 | |
| .04 | 1. | −4.0 | −3.5 | .23 | |
| .08 | 1. | −2.5 | −2.1 | .18 | |

TABLE 6
THE EFFECTS OF A DEPLETION ALLOWANCE

| a | $\Gamma_1$ | $\overline{R}$ Cum. Ore Extraction ($10^6$ tons) | $\overline{Q}$ Cum. Metal Production ($10^6$ lbs) | $\overline{Y}$ Average Yield (%) | $D_\Gamma$ ($10^6$ \$) |
|---|---|---|---|---|---|
| .00 | −.05 | 139 | 3046 | 1.092 | 34.5 |
| .04 | −.05 | 155 | 3446 | 1.112 | 31.8 |
| .08 | −.05 | 162 | 3685 | 1.140 | 34.0 |
| .00 | −.10 | 143 | 3133 | 1.095 | 66.6 |
| .04 | −.10 | 158 | 3514 | 1.115 | 61.1 |
| .08 | −.10 | 163 | 3738 | 1.143 | 65.6 |
| a | $\Gamma_1$ | %Δ Cum. Ore Extraction | %Δ Cum. Metal Production | %Δ Average Yield | |
| .00 | −.05 | 2.2 | 3.2 | .65 | |
| .04 | −.05 | 2.0 | 2.2 | .27 | |
| .08 | −.05 | 1.3 | 1.6 | .62 | |
| .00 | −.10 | 5.2 | 6.2 | .92 | |
| .04 | −.10 | 4.0 | 4.2 | .54 | |
| .08 | −.10 | 1.9 | 3.1 | .88 | |

TABLE 7
THE EFFECTS OF PRICE CONTROLS

| | Anticipated | | |
|---|---|---|---|
| a | $\overline{R}$ Cum. Ore Extraction ($10^6$ tons) | $\overline{Q}$ Cum. Metal Production ($10^6$ lbs) | $\overline{Y}$ Average Yield (%) |
| .04 | 149 | 3287 | 1.102 |
| .08 | 159 | 3558 | 1.116 |
| a | %Δ Cum. Ore Extraction | %Δ Cum. Metal Production | %Δ Average Yield |
| .04 | −2.0 | −2.5 | − .66 |
| .08 | − .60 | −1.9 | −1.7 |
| | Unanticipated | | |
| a | $\overline{R}$ Cum. Ore Extraction ($10^6$ tons) | $\overline{Q}$ Cum. Metal Production ($10^6$ lbs) | $\overline{Y}$ Average Yield (%) |
| .04 | 145 | 3207 | 1.104 |
| .08 | 148 | 3303 | 1.116 |
| a | %Δ Cum. Ore Extraction | %Δ Cum. Metal Production | %Δ Average Yield |
| .04 | −4.6 | −4.9 | − .50 |
| .08 | −7.5 | −8.9 | −1.73 |

anticipated control results in an increase in extraction in the periods immediately after it is instituted. This happens because producers were anticipating higher prices in later periods that will not be realized. When the control is placed, they therefore have less incentive to postpone extraction.

## VI. SUMMARY AND CONCLUSIONS

The change in the intertemporal supply of exhaustible resources due to tax policy is an important subject, both because of the prevalence of taxes, subsidies, and controls and because of their magnitude. These effects, however, are only poorly understood. The way in which changes in taxes affect extraction and processing profiles depends on the type of tax, where it is placed, how it affects input and output prices, and on the technology of extraction and processing.

In this paper, some of the restrictive assumptions that earlier studies rely on are relaxed. Because it is difficult to obtain unambiguous theoretical results that are independent of the restrictive assumptions, numerical techniques are used.

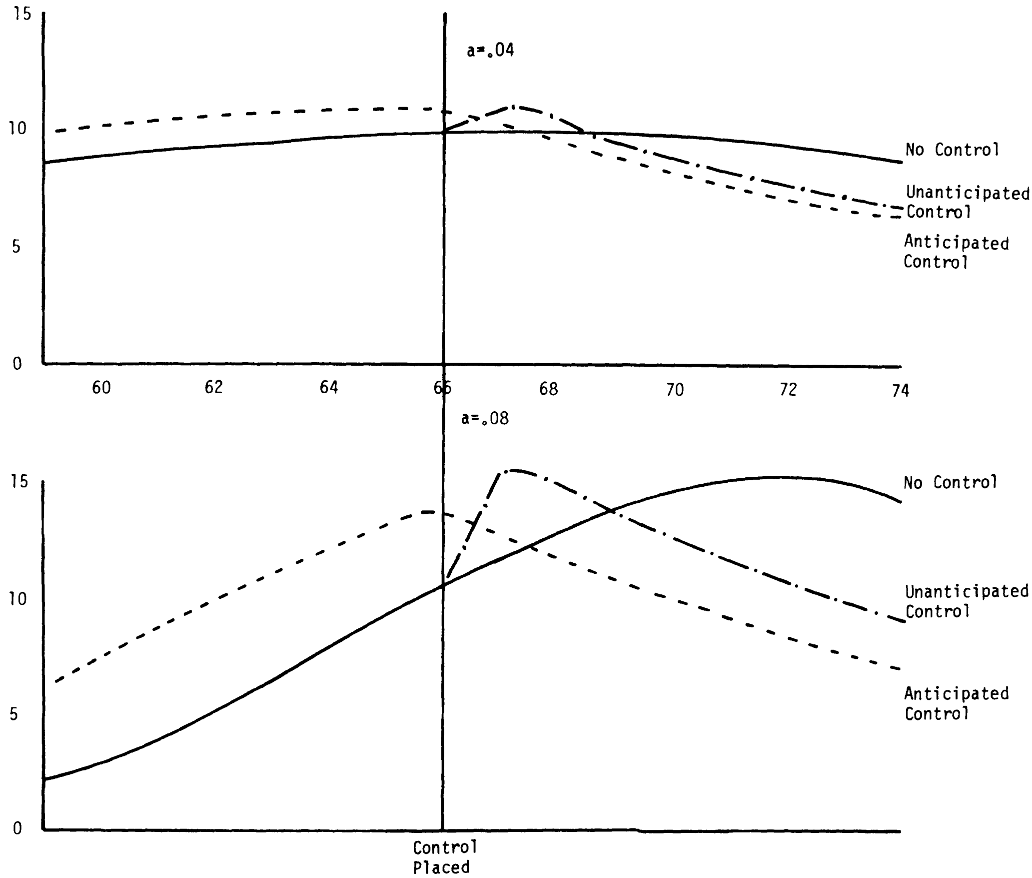


FIGURE 4
THE EFFECTS OF PRICE CONTROLS

A variable-profit function for a particular copper-producing firm is estimated. This profit function is then used in numerical simulations that determine optimal extraction and processing paths under various tax and subsidy policies. Using this technique it is possible to estimate both the direction and the magnitude of tax-policy distortions in a fairly realistic setting.

The principal effects of taxation that are observed are tilting of the extraction path, changes in cumulative ore extraction, and changes in cumulative metal production. In many simple exhaustible-resource models (with constant marginal extraction costs that do not shift up with cumulative production, for example), it is always optimal to extract the entire resource. Only the first effect (tilting) can therefore be observed. In a more realistic model, however, the second and third effects predominate. In the simulations reported here, some tilting is observed. At high rates of output-price appreciation, royalties (depletion allowances) lead to higher (lower) rates of extraction in earlier years which are later reversed. Under these circumstances, the imposition of the royalty has the opposite effect from what is often assumed. It is interesting to note that royalties (depletion allowances) never lead to unambiguous conservation (depletion), as is often supposed.

When the extracted resource is the only input to enter the cost function, percent changes in cumulative ore and metal production must be equal. When the ore is combined with other inputs, however, taxes and subsidies can change extraction rates and processing intensities in opposite ways. For example, the imposition of a royalty causes both cumulative ore extraction and the intensity of processing to fall and therefore leads to a larger decrease in metal production than in ore extraction. In contrast, the imposition of a severance tax causes cumulative ore extraction to fall while processing intensity rises. The latter occurs because ore has become expensive relative to other inputs. Unlike a royalty, a severance tax therefore results in a smaller decrease in metal production than in ore extraction.

Tax-policy effects depend on whether or not the imposition of the policy is anticipated. For example, with price controls it is found that cumulative ore and metal production fall more when the control is unanticipated than if producers are aware that controls will be placed. In addition, an interesting phenomenon occurs when controls are placed unexpectedly. Extraction rates actually increase in the period immediately following the imposition of the control. This happens because producers were anticipating higher prices in future years which will not be realized. When the control is placed, they therefore have less incentive to postpone extraction.

The magnitudes of tax-policy distortions observed here are sizable. For example, the imposition of a 10% royalty causes an 8% reduction in cumulative metal production resulting in a distortion valued at $78 million for a single mine.

The combination of large and often counterintuitive effects of tax policy implies a need for a better understanding of their ramifications. The model developed here is a useful research and teaching tool. Many other simulation experiments can be performed easily and it is possible to use the model to investigate the consequences of any proposal to change the way in which extractive firms are taxed. For example, taxation rates need not be constant but can be functions of some other variable (e.g., profits) or can vary with time. In addition, the different ways in which producers react to anticipated and unanticipated taxes can be assessed.

Given the importance of the problem, there is need for further research in the area. One obvious extension to the work reported here is the endogenization of the planning horizon—determination of the optimal lifetime for the mine as a function of tax policy. In fact, such an investigation is underway. Another important issue that has been neglected is the way in which market structure affects tax-policy distortions. For the monopoly case, qualitative results should be similar to those reported here. The magnitudes of the distortions, however, could be very different under monopoly and competitive conditions. Finally, I have concentrated my efforts on extraction and processing profiles for an existing mine and have ignored the important issue of how exploration for and investment in new mines changes with changes in tax policy.

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